Quiz #5 Solution

Date: 03/11/2013 Name: ____________________

NOTE: You must show all work to earn credit.

1. (6 points) Given $f(x, y, z) = \sqrt{3x^2 + y^2 - 2z^2}$. Evaluate f_x, f_y, f_z at $(1, -2, 1)$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[(3x^2 + y^2 - 2z^2)^{1/2}\right] = \frac{1}{2}(3x^2 + y^2 - 2z^2)^{-1/2}(3x^2 + y^2 - 2z^2)'$$
$$= \frac{1}{2}(3x^2 + y^2 - 2z^2)^{-1/2}(6x) = 3x(3x^2 + y^2 - 2z^2)^{-1/2}.$$

Similarly, $\frac{\partial f}{\partial y} = y(3x^2 + y^2 - 2z^2)^{-1/2}$, and $\frac{\partial f}{\partial z} = -2z(3x^2 + y^2 - 2z^2)^{-1/2}$.

Therefore, $\frac{\partial f}{\partial x}(1, -2, 1) = 3(3 + 4 - 2)^{-1/2} = 3/\sqrt{5}$.
Similarly, $\frac{\partial f}{\partial y}(1, -2, 1) = -2(3 + 4 - 2)^{-1/2} = -2/\sqrt{5}$, and
$\frac{\partial f}{\partial z}(1, -2, 1) = -2(3 + 4 - 2)^{-1/2} = -2/\sqrt{5}$.

2. (9 points) Given $f(x, y) = ye^x$. (a) Evaluate $f(2, 1)$ and $f(2.1, 1.05)$ and calculate Δz, and (b) Use the total differential dz to approximate Δz.

Solution:

(a) $f(2, 1) = 1e^2 \approx 7.389056$,
$f(2.1, 1.05) = 1.05e^{2.1} \approx 8.574478$, and
$\Delta z = f(2.1, 1.05) - f(2, 1) = 1.05e^{2.1} - 1e^2 \approx 1.185422$.

(b) $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = ye^x dx + e^x dy$.
Notice that $x = 2, y = 1, dx = 0.1$, and $dy = 0.05$.
$dz = ye^x dx + e^x dy = (1)(e^2)(0.1) + (e^2)(0.05) \approx 1.108358$.

Note: By the way, we can use the formula: $f(x + dx, y + dy) \approx f(x, y) + dz$ to find
$f(2.1, 1.05) \approx f(2, 1) + 1.108358 \approx 7.389056 + 1.108358 = 8.497415$
which is 0.077064 off the target: 8.54478.

3. (7 points) Electrical power P is given by $P = E^2/R$, where E is voltage and R is resistance. Approximate the maximum percent error in calculating power if 120 volts is applied to a 2000-ohm resistor and the possible percent errors in measuring E and R are 3% and 4%, respectively.

Solution:

Given $P = E^2/R, E = 120, R = 2000$. So, $P = 14400/2000 = 7.2$.

Notice that we have:

$dE = 120 \times (\pm 3\%) = \pm 3.6, dR = 2000 \times (\pm 4\%) = \pm 80$. Also,

$$dP = \frac{\partial P}{\partial E} dE + \frac{\partial P}{\partial R} dR = \frac{2E}{R} dE - \frac{E^2}{R^2} dR$$
$$= \frac{240}{2000}(\pm 3.6) - \frac{14400}{4000000}(\pm 80) = \pm 0.432 - (\pm 0.288)$$

This value dP is between -0.72 and 0.72. Therefore, the maximum percent error is

$$\left|\frac{dP}{P}\right| = \frac{0.72}{7.2} = 0.1 = 10\%.$$

4. (8 points) Given $w = (x - y)\sin(y - x)$, $x = u - v$, and $y = v - u$. Find $\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v}$.

Solution:

We shall use the chain rules to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}$$
$$= [\sin(y - x) + (x - y)\cos(y - x)(-1)](1) + [-\sin(y - x) + (x - y)\cos(y - x)](-1)$$
$$= -2(x - y)\cos(y - x).$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}$$
$$= [\sin(y - x) + (x - y)\cos(y - x)(-1)](-1) + [-\sin(y - x) + (x - y)\cos(y - x)](1)$$
$$= 2(x - y)\cos(y - x).$$

Therefore, $\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = -2(x - y)\cos(y - x) + 2(x - y)\cos(y - x) = 0$.