Quiz #2 Solution

Date: 01/29/2013 Name: ____________________

NOTE: You must show all work to earn credit.

1. (8 points) Given $\vec{u} = \langle 2, 1, 2 \rangle$ and $\vec{v} = \langle 0, 3, 4 \rangle$.
   (a) Find the projection of $\vec{u}$ onto $\vec{v}$;
   (b) Find the vector component of $\vec{u}$ orthogonal to $\vec{v}$.

Solution:

Given $\vec{u} = \langle 2, 1, 2 \rangle$ and $\vec{v} = \langle 0, 3, 4 \rangle$.

(a) $\vec{w}_1 = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v} = \frac{11}{25}\langle 0, 3, 4 \rangle = \langle 0, \frac{33}{25}, \frac{44}{25} \rangle$

(b) $\vec{w}_2 = \vec{u} - \vec{w}_1 = \langle 2, 1, 2 \rangle - \langle 0, \frac{33}{25}, \frac{44}{25} \rangle = \langle 2, -\frac{8}{25}, \frac{6}{25} \rangle$

2. (8 points) Given $A(1,1,1)$, $B(2,3,4)$, $C(6,5,2)$, and $D(7,7,5)$. Verify that the points are the vertices of a parallelogram, and find its area.

Solution:

Given $A(1,1,1)$, $B(2,3,4)$, $C(6,5,2)$, and $D(7,7,5)$. We can find the vectors: $\overrightarrow{AB} = \langle 1, 2, 3 \rangle$, $\overrightarrow{AC} = \langle 5, 4, 1 \rangle$, $\overrightarrow{CD} = \langle 1, 2, 3 \rangle$, and $\overrightarrow{BD} = \langle 5, 4, 1 \rangle$.

Obviously $\overrightarrow{AB} = \overrightarrow{CD}$, and $\overrightarrow{AC} = \overrightarrow{BD}$. So the figure $ABCD$ is a parallelogram. As $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides, we can use the cross product of these two vectors to find the area of the parallelogram.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{vmatrix} = -10\vec{i} + 14\vec{j} - 6\vec{k}$$

Therefore, the area

$$A = \|\overrightarrow{AB} \times \overrightarrow{AC}\| = \|-10\vec{i} + 14\vec{j} - 6\vec{k}\| = \sqrt{332} = 2\sqrt{83} \approx 18.22$$

3. (7 points) Find the distance between the point $Q(2,8,4)$ and the plane $2x+y+z=5$.

Solution:

Given the point $Q(2,8,4)$ and the plane $2x+y+z=5$.

A normal vector of the plane is: $\vec{n}=\langle 2,1,1\rangle$ (Note: you can choose any vector that is parallel to this one). Obviously $P(0,0,5)$ is in the plane (Note: you can choose any other point in the plane, as long as it satisfies the equation of the plane).

Now $\overrightarrow{PQ}=\langle 2,8,-1\rangle$. Using the distance formula between a point and a plane,

$$D=\frac{\left|\overrightarrow{PQ}\cdot\vec{n}\right|}{\|\vec{n}\|}=\frac{|\langle 2,8,-1\rangle\cdot\langle 2,1,1\rangle|}{\|\langle 2,1,1\rangle\|}=\frac{11}{\sqrt{6}}=\frac{11\sqrt{6}}{6}$$

Therefore, the distance between the point $Q(2,8,4)$ and plane $2x+y+z=5$ is $\frac{11\sqrt{6}}{6}$.

4. (7 points) Find the distance between point $P(1,-2,4)$ and the line given by equations $x=2t$, $y=t-3$, $z=2t+2$.

Solution:

Obviously $\vec{u}=\langle 2,1,2\rangle$ is the direction vector for the line. Let $t=0$, we get a point on the line $Q(0,-3,2)$. Then $\overrightarrow{QP}=\langle 1,1,2\rangle$.

Let us calculate $\overrightarrow{QP}\times\vec{u}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 2\\ 2 & 1 & 2\end{vmatrix}=\langle 0,2,-1\rangle$.

Using the distance formula between a point and a line,

$$D=\frac{\left\|\overrightarrow{QP}\times\vec{u}\right\|}{\|\vec{u}\|}=\frac{\|\langle 0,2,-1\rangle\|}{\|\langle 2,1,2\rangle\|}=\frac{\sqrt{5}}{\sqrt{9}}=\frac{\sqrt{5}}{3}$$

Therefore, the distance between the point $P(1,-2,4)$ and the given line is $\frac{\sqrt{5}}{3}$.