Quiz #1 Solution

Date: 01/22/2013 Name: _____

NOTE: You must show all work to earn credit.

1. (9 points) Three forces with magnitudes of 400 pounds, 280 pounds, and 350 pounds act on an object at angles of $-30°$, $45°$, and $135°$, respectively, with the positive x-axis. Find the direction and magnitude of the resultant force.

Solution:

Let $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ be those three forces. Then

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 400(\cos(-30°)\vec{\imath} + \sin(-30°)\vec{\jmath}) + 280(\cos 45°\vec{\imath} + \sin 45°\,\vec{\jmath}) + 350(\cos 135°\vec{\imath} + \sin 135°\,\vec{\jmath})$$
$$= 200\sqrt{3}\vec{\imath} - 200\vec{\jmath} + 140\sqrt{2}\vec{\imath} + 140\sqrt{2}\vec{\jmath} - 175\sqrt{2}\vec{\imath} + 175\sqrt{2}\vec{\jmath}$$
$$= \left(200\sqrt{3} - 35\sqrt{2}\right)\vec{\imath} + \left(-200 + 315\sqrt{2}\right)\vec{\jmath}$$

The magnitude of the resultant force is $\sqrt{\left(200\sqrt{3} - 35\sqrt{2}\right)^2 + \left(-200 + 315\sqrt{2}\right)^2} \approx 385.2$ pounds.

The direction angle $\theta = \arctan\left(\frac{-200+315\sqrt{2}}{200\sqrt{3}-35\sqrt{2}}\right) \approx 0.6908 \approx 39.6°$

2. (7 points) Given $\vec{u} = \langle 4, 9\rangle$, and $\vec{v} = \langle 2, -5\rangle$. Find
   (a) $\frac{2}{3}\vec{u}$, (b) $\vec{v} - \vec{u}$, (c) $2\vec{u} + 5\vec{v}$.

Solution: Given $\vec{u} = \langle 4, 9\rangle$, and $\vec{v} = \langle 2, -5\rangle$.
(a) $\frac{2}{3}\vec{u} = \frac{2}{3}\langle 4, 9\rangle = \langle \frac{8}{3}, 6\rangle$
(b) $\vec{v} - \vec{u} = \langle 2, -5\rangle - \langle 4, 9\rangle = \langle -2, -14\rangle$
(c) $2\vec{u} + 5\vec{v} = 2\langle 4, 9\rangle + 5\langle 2, -5\rangle = \langle 18, -7\rangle$

3. (7 points) Determine which of the vectors is/are parallel to $\vec{z} = \langle 3, 2, -5\rangle$:
   (a) $\langle -6, -4, 10\rangle$, (b) $\langle 2, \frac{4}{3}, -\frac{10}{3}\rangle$, (c) $\langle 6, 4, 10\rangle$, (d) $\langle 1, -4, 2\rangle$.

Solution: (a) and (b) are parallel to $\vec{z} = \langle 3, 2, -5\rangle$ since
(a) $\langle -6, -4, 10\rangle = -2\langle 3, 2, -5\rangle$
(b) $\langle 2, \frac{4}{3}, -\frac{10}{3}\rangle = \frac{2}{3}\langle 3, 2, -5\rangle$
(c) $\langle 6, 4, 10\rangle \neq c\langle 3, 2, -5\rangle$
(d) $\langle 1, -4, 2\rangle \neq c\langle 3, 2, -5\rangle$

4. (7 points) Find the angle between two vectors $\vec{u} = \langle 1, 1, 1\rangle$ and $\vec{v} = \langle 2, 1, -1\rangle$.

Solution: Given $\vec{u} = \langle 1, 1, 1\rangle$, and $\vec{v} = \langle 2, 1, -1\rangle$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{2}{\sqrt{3}\sqrt{6}} = \frac{\sqrt{2}}{3}.$$

Therefore, $\theta = \arccos\frac{\sqrt{2}}{3} \approx 61.9°$