Lecture 37

15.8. Stokes' Theorem

Goals: (1) Understand and use Stokes' Theorem.

(2) Use curl to analyze the motion of rotating liquid.

Questions:

- How to find the curl of a vector field?
- What is the first alternative form of Green's Theorem?

15.8.1. **Stokes' Theorem**

Recall that the first alternative form of Green's Theorem (p. 1098) gives the relationship between a line integral and a double integral. In an analogous way, Stokes' Theorem is a 2nd higher-dimension of Green's Theorem.

(1) Formula:

Let S be an oriented surface with unit normal vector $\vec{N}$, bounded by a piecewise smooth simple closed curve C. If $\vec{F}$ is a vector field whose component functions have continuous partial derivatives on an open region containing S and C, then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \left(\overrightarrow{\text{curl}}\, \vec{F}\right) \cdot \vec{N} \quad dS$$

(2) Example 1: Using Stokes' Theorem (pp. 1133)
Try exercises 11-20

(3) Example 2: Verifying Stokes' Theorem (p. 1134)
Try exercises 7-10

15.8.2. **Physical interpretation of curl**

$\overrightarrow{\text{curl}}\, \vec{F}(x, y, z) \cdot \vec{N} \quad$ = rotation of $\vec{F}$ about $\vec{N} \quad$ at (x, y, z). See Figure 15.67 (p. 1135). The dot product will be maximum if the curl of vector field is in the same direction as $\vec{N} \quad$.

The formula $\int_C \vec{F} \cdot d\vec{r} = \iint_S \left(\overrightarrow{\text{curl}}\, \vec{F}\right) \cdot \vec{N} \quad dS$ says that the collective measure of this *rotational* tendency taken over the entire surface S (surface integral) is equal to the tendency of a fluid to *circulate* around the boundary C (line integral).

15.8.3. **Homework Set #37**

- Read 15.8 (pages 1132-1136).
- Do exercises on pages 1136-1137:
  1, 3, 5, 7, 9, 11, 13, 15, 17, 19