Lecture 36

15.7. Divergence Theorem

Goals: (1) Understand and use the Divergence Theorem.

(2) Use the Divergence Theorem to calculate flux (skipped).

Question:

- What is the alternative form of Green's Theorem related to divergence?

15.7.1. **Divergence Theorem (Gauss-Ostrogradsky Theorem)**

Recall that the alternative form of Green's Theorem gives the relationship between a line integral and a double integral. In an analogous way, the Divergence Theorem gives the relationship between a trip integral over a solid region Q and a surface integral over the surface of Q.

(1) Formula:
Let Q be a solid region bounded by a *closed* surface S oriented by a unit normal vector $\vec{\boldsymbol{N}}$ directed outward from Q. If $\vec{\boldsymbol{F}}(x, y, z) = M\vec{i} + N\vec{j} + P\vec{k}$ is a vector field whose component functions have continuous partial derivatives in Q, then

$$\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{N}} dS = \iiint_Q \operatorname{div} \vec{\boldsymbol{F}}\, dV = \iiint_Q \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}\right) dV$$

(2) Examples of closed surfaces:

- Spheres, Ellipsoids
- Cubes
- Tetrahedrons
- The combinations of above

(3) Examples 1, 3: Using the Divergence Theorem (pp. 1126, 1128)
Try exercises 7-18

(4) Example 2: Verifying the Divergence Theorem (p. 1127)
Try exercises 1-6

15.7.2. **Flux and the Divergence Theorem**

Skipped.

15.7.3. **Homework Set #36**

- Read 15.7 (pages 1124-1130).

- Do exercises on pages 1130-1131:
  1, 3, 5, 7, 9, 11, 13, 15