Lecture 33

15.4. Green's Theorem

Goals: (1) Use Green's Theorem to evaluate a line integral.

(2) Use alternative forms of Green's Theorem.

Questions:

- What is a conservative vector field? How to evaluate a double integral?
- How to evaluate a line integral?

15.4.1. **Green's Theorem**

This theorem states that the value of a double integral over a *simply connected plane region* R is determined by the value of a line integral around the boundary of R.

(1) Green's theorem:

Let R be a simply connected region with a piecewise smooth boundary C, oriented counterclockwise (that is, C traversed once so that the region R always lies to the left). If M and N have continuous partial derivatives in an open region containing R, then

$$\int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA.$$

(2) Example 1: using Green's theorem to calculate line integral (p. 1094)
Try exercises 1-10

(3) Example 2: using Green's theorem to calculate work (p. 1095)
Try exercises 21-24

(4) Example 3: using Green's theorem to calculate line integral with conservative vector field (p. 1095)
Example 4: using Green's theorem for a piecewise smooth curve (p. 1096)
Try exercises 11-20

(5) Example 5: using line integral to find area! (p. 1097)
Try exercises 25-28

(6) Example 6: using Green's theorem for a region with a hole (p. 1097)

15.4.2. **Alternative forms of Green's Theorem**

(1) First alternative form:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R \left(\overrightarrow{\text{curl}}\, \vec{F}\right) \cdot \vec{k}\, dA$$

(2) Second alternative form:

$$\int_C \vec{F} \cdot \vec{N}\, ds = \iint_R \text{div}\, \vec{F} dA$$

where $\vec{N}$ is the (outward) unit normal vector of $\vec{r}$.

15.4.3. **Homework Set #33**

- Read 15.4 (pages 1093-1099).
- Do exercises on pages 1099-1101:
  1, 3, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27