# Lecture 29

## 14.7. Triple Integrals in Cylindrical and Spherical Coordinates

**Goals:** (1) Write and evaluate a triple integral in cylindrical coordinates.

(2) Write and evaluate a triple integral in spherical coordinates.

Questions:

- What are the conversion equations for cylindrical coordinates?
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$
- What are the conversion equations for spherical coordinates?
$$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$$

### 14.7.1. **Triple integrals in cylindrical coordinates**

Note: $dV = rdz\,dr\,d\theta$ for cylindrical coordinates. See Figure 14.63 (p. 1038)

(1) $r$-simple case:

If $Q$ is a $r$-simple, then

$$\iiint_Q f(x,y,z)dV = \int_{\theta_1}^{\theta_2}\int_{g_1(\theta)}^{g_2(\theta)}\int_{h_1(r\cos\theta, r\sin\theta)}^{h_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$$

(2) $\theta$-simple case:

If $Q$ is a $\theta$-simple, then

$$\iiint_Q f(x,y,z)dV = \int_{r_1}^{r_2}\int_{g_1(r)}^{g_2(r)}\int_{h_1(r\cos\theta, r\sin\theta)}^{h_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,d\theta\,dr$$

(3) Example 1: finding volume by cylindrical coordinates (p. 1039)
Try exercises 1-3, 9-10, 17-22

(4) Example 2: finding mass by cylindrical coordinates (p. 1040)
Try exercises 23-24

(5) Example 3: finding moment of inertia by cylindrical coordinates (p. 1040)
Try exercises 29-30, 31-32

14.7.2. **Triple integrals in spherical coordinates**

Note: $dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ for spherical coordinates. See Figure 14.68 (p. 1041)

(1) A spherical block case:
If $Q$ is a spherical block determined by
$\{(\rho, \theta, \phi): \rho_1 \le \rho \le \rho_2, \theta_1 \le \theta \le \theta_2, \phi_1 \le \phi \le \phi_2\}$. Then

$$\iiint_Q f(x,y,z)dV$$

$$= \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \int_{\rho_1}^{\rho_2} f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi) \, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

(2) Example 4: finding volume in spherical coordinates (p. 1042)
Try exercises 4-6, 11-12, 33-36

(3) Example 5: finding the center of mass (p. 1042)
Try exercises 39-40.

14.7.3. **Homework Set #29**

- Read 14.7 (pages 1038-1042).
- Do exercises on pages 1043-1044:
  1, 3, 5, 9, 11, 13, 15, 17, 19, 21, 33, 37