Lecture 23

14.1. Iterated Integrals and Area in the Plane

Goals: (1) Evaluate an iterated integral.

(2) Use an iterated integral to find the area of a plane region.

Questions:

- What are the common techniques for integration?
- How to find the area of a region using definite integral?

14.1.1. **Iterated integrals**

(1) Two single integrals:

- x is the variable, y is fixed

$$\int_{h_1(y)}^{h_2(y)} f_x(x,y)dx = f(x,y)]_{h_1(y)}^{h_2(y)} = f(h_2(y),y) - f(h_1(y),y)$$

- y is the variable, x is fixed

$$\int_{g_1(x)}^{g_2(x)} f_y(x,y)dy = f(x,y)]_{g_1(x)}^{g_2(x)} = f\big(x,g_2(x)\big) - f\big(x,g_1(x)\big)$$

- Example 1 (p. 985)
- Try Exercises 1-10

(2) Iterated integrals:

- y first (inside), x second (outside)

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f_y(x,y)dy\,dx$$

- x first (inside), y second (outside)

$$\int_c^d \int_{h_1(y)}^{h_2(y)} f_x(x,y)dx\,dy$$

- Example 2 (p. 985)
- Try Exercises 11-30

14.1.2. **Area of a plane region**

(1) Vertically simple region (see Figure 14.2)

$$A = \int_a^b [g_2(x) - g_1(x)]\,dx = \int_a^b \int_{g_1(x)}^{g_2(x)} dy\,dx$$

- Examples 3, 4 (p. 987)
  Try exercises 35-38, 39-46.

(2) Horizontally simple region (see Figure 14.3)

$$A = \int_c^d [h_2(y) - h_1(y)]\, dy = \int_c^d \int_{h_1(y)}^{h_2(y)} dx\, dy$$

- Example 5 (p. 988)
  Try exercises 47-54, 55-58, 61-64
- Note: both orders should result in the same area, though one order may be much easier to evaluate.

(3) Almost-simple region: sum of two simple regions

$$A = A_1 + A_2$$

- Example 6 (p. 989)
  Try exercises 59, 60

14.1.3. **Homework Set #23**

- Read 14.1 (pages 984-989).
- Do exercises on pages 990-991:
  5, 7, 9, 13, 15, 17, 19, 21, 23, 25, 31, 33, 35, 37, 41, 43, 45, 47, 49, 51, 53, 55, 59, 61, 63, 67, 69, 71, 87, 88