Lecture 21

13.9. Applications of Extrema of Functions of Two Variables

Goals: (1) Solve optimization problems involving functions of several variables.

(2) Use the method of least squares.

Question:

- How to find relative extrema of a function?

13.9.1. **Applied optimization problems**

(1) Volume, Area, Distance
- Example 1 (p. 962)
- Exercises 1-4
- Exercises 10-12, 13-14

(2) Profit, Revenue, Cost
- Example 2 (p. 963)
- Exercises 15-16

13.9.2. **The method of least squares**

(1) Definition
- Given n points: $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, and a model (function) $y = f(x)$. Then

$$S = \sum_{i=1}^{n} [f(x_i) - y_i]^2$$

is called the ***sum of the squared errors***.
- Given n points: $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. Let

$$a = \frac{n(\sum_{i=1}^{n} x_i y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2}, \text{and } b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a \sum_{i=1}^{n} x_i\right).$$

Then $y = ax + b$ is called the ***least squares regression line*** (LSRL) for those points.

Note: We use those points to calculate the slope and intercept of the line! (See Figure 13.76) If $\sum_{i=1}^{n} x_i = 0$, then we have special LSRL $y = ax + b$ where

$$a = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}, \text{and } b = \frac{1}{n}\sum_{i=1}^{n} y_i.$$

- Models:
  + linear model, e.g., $y = 1.8566x - 5.0246$
  + quadratic model, e.g., $y = 0.1996x^2 - 0.7281x + 1.3749$

(2) Data

- Example 3 (p. 966)

13.9.3. **Homework Set #21**

- Read 13.9 (pages 962-966).
- Do exercises on pages 966-969:
  1, 3, 5, 7, 9, 13, 15, 19, 31, 33, 35, 45