Lecture 13

13.1. Introduction to Functions of Several Variables

Goals: (1) Understand the notation for a function of several variables.

(2) Sketch the graph of a function of two variables.

(3) Sketch level curves for a function of two variables.

(4) Sketch level surfaces for a function of three variables.

Questions:

- What is the domain and range of a function $y = f(x)$?

13.1.1. **Functions of several variables**

(1) Definition: function of two variables.
$z = f(x, y)$, x, y are independent variables, z is dependent variable.

(2) Operations:

Sum/Difference	$(f \pm g)(x, y) = f(x, y) \pm g(x, y)$
Product	$(fg)(x, y) = f(x, y)g(x, y)$
Quotient	$(f/g)(x, y) = f(x, y)/g(x, y), g(x, y) \neq 0$
Composition	$(f \circ g)(x, y) = f\big(g(x, y)\big)$

(3) Example 1: finding the domain of the function (p. 887).
- Try exercises 19-30

13.1.2. **The graph of a function of two variables**

(1) The graph of a function $z = f(x, y)$ is a surface in space whose projection onto the xy-plane is the domain D.

(2) How to sketch a surface in space by hand?
Use traces in planes parallel to the coordinate planes.

(3) Example 2: describing the graph of a function of two variables (p. 888).
- Try exercises 33-40

13.1.3. **Level curves**

(1) Another way to visualize a function $z = f(x, y)$ is to use a *scalar field* that is characterized by *level curves* $f(x, y) = c$ (contour lines, or equipotential lines).

(2) Contour maps are commonly used to depicts the variation of z with respect to x and y by the spacing between level curves. Much space between level curves indicates that z is changing slowly, whereas little space indicates a rapid change in z.

(3) Examples 3, 4: sketching contour maps (p. 890).

$$f(x,y) = \sqrt{64 - x^2 - y^2}$$
$$z = y^2 - x^2$$

- Try exercises 45-48, 49-56

13.1.4. **Level surfaces**

(1) The concept of a level curve can be extended by one dimension to define a level surface. Given a function $z = f(x,y,z)$, the graph of the equation $f(x,y,z) = c$ is called a *level surface*.

(2) Example 6: sketching level surfaces (p. 892).

$$f(x,y,z) = 4x^2 + y^2 + z^2$$

- Try exercises 69-74

13.1.5. **Homework Set #13**

- Read 13.1 (pages 886-893).
- Do exercises on pages 894-897:
  5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 33, 35, 37, 49, 53, 55, 69, 71, 81, 89-92