Lecture 8

12.1. Vector-Valued Functions

Goals: (1) Analyze and sketch a space curve given by a vector-valued function.

(2) Extend the concepts of limits and continuity to vector-valued functions.

Questions:

- What is a plane curve?

12.1.1. **Space curves**

A ***space curve*** C is the set of all ordered triples $\left(f(t), g(t), h(t)\right)$ where $x = f(t)$, $y = g(t)$, and $z = h(t)$ are continuous functions on an interval I.

- Example: $x = 2t$, $y = 3t^2$ and $z = e^t$.

12.1.2. **Vector-valued functions**

(1) Definition: Vector-valued function is (See Figure 12.1)

- a function (in plane) of the form:

$$\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} = \langle f(t), g(t) \rangle$$

- a function (in space) of the form:

$$\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k} = \langle f(t), g(t), h(t) \rangle$$

<u>Note</u>: If not given, the domain of $\vec{r}(t)$ is the intersection of domains of $f(t), g(t), h(t)$. Try exercises 1-8.

(2) Examples 1, 2: Given vector-valued functions, sketching curves.

- Try exercises 27-42.

(3) Examples 3, 4: Given graphs by rectangular equations, find vector-valued functions.

- Try exercises 49-56, 59-66.

12.1.3. **Limits and continuity**

(1) Definition of limit of a vector-valued function (plane or space):

- If $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath}$, then $\lim_{t\to a} \vec{r}(t) = [\lim_{t\to a} f(t)]\vec{\imath} + [\lim_{t\to a} g(t)]\vec{\jmath}$, provided that the component limits exist.
- If $\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k}$, then $\lim_{t\to a} \vec{r}(t) = [\lim_{t\to a} f(t)]\vec{\imath} + [\lim_{t\to a} g(t)]\vec{\jmath} + [\lim_{t\to a} h(t)]\vec{k}$, provided that the component limits exist.

(2) Definition of continuity of a vector-valued function:

- A vector-valued function $\vec{r}$ is continuous at the point given by $t = a$ if the limit $\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$.

- A vector-valued function $\vec{r}$ is continuous on an interval I if it is continuous at every point in the interval.

(3) Example 5: Continuity of vector-valued functions.

- Try exercises 69-74, 75-80

12.1.4. **Homework Set #8**

- Read 12.1 (pages 834-838).
- Do exercises on pages 839-841:
  1, 3, 5, 7, 9, 11, 13, 21-24, 27, 29, 31, 35, 39, 49, 51, 59, 61, 63, 65, 69, 71, 73, 75, 77, 79, 93-96