Lecture 7

11.7. Cylindrical and Spherical Coordinates

Goals: (1) Use cylindrical coordinates to represent surfaces in space.

(2) Use spherical coordinates to represent surfaces in space.

Questions:

- What is the polar coordinate system?

11.7.1. **Cylindrical coordinates in space**

(1) In a cylindrical coordinate system, a point P in space can be represented by an ordered triple (r, θ, z).

- (r, θ) is a polar representation of the projection of P in the xy-plane.
- z is the directed distance from (r, θ) to P.

See Figure 11.66.

(2) How to convert from cylindrical to rectangular coordinates?

- $x = r\cos\theta\,, y = r\sin\theta\,, z = z$.

Note: The conversion is very simple!

(3) How to convert from rectangular to cylindrical coordinates?

- $r^2 = x^2 + y^2, \tan\theta = \frac{y}{x}, z = z$.

Note: $(0, 0, 0)$ is called the pole. The representation is not unique…

(4) Examples 1, 2: Coordinates conversion.

- Try exercises 1-12.

11.7.2. **Cylindrical surfaces in cylindrical coordinate system**

(1) Special cylindrical surfaces

- Vertical plane: $\theta = c$
- Horizontal plane: $z = c$
- Cylinder: $r = c$
- Paraboloid: $r = c\sqrt{z}$
- Cone: $r = cz$
- Hyperboloid: $r^2 = cz^2 + d$

Note: See Figures 11.69 and 11.70

(2) Examples 3, 4: equation conversion between two systems.

- Try exercises 13-28.

11.7.3. **Spherical coordinates in space**

(1) In a spherical coordinate system, a point P in space can be represented by an ordered triple (ρ, θ, φ).

- ρ is the distance between P and the origin, where $\rho \geq 0$.

- θ is the same angle used in the cylindrical coordinates for $r \geq 0$.
- φ is the angle between the positive z-axis and the line segment $\overline{OP}$, where $0 \leq \varphi \leq \pi$.

See Figure 11.75.

(2) How to convert from spherical to rectangular coordinates?

$$x = \rho \sin\varphi \cos\theta \, , y = \rho \sin\varphi \sin\theta \, , z = \rho \cos\varphi$$

(3) How to convert from rectangular to spherical coordinates?

$$\rho^2 = x^2 + y^2 + z^2, \tan\theta = \frac{y}{x}, \varphi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

(4) Example 5: Rectangular-to-spherical conversion.

- Try exercises 29-34.

11.7.4. **Coordinates conversion between cylindrical and spherical systems**

(1) Spherical to cylindrical.

$$r^2 = \rho^2 \sin^2\varphi \, , \theta = \theta, z = \rho \cos\varphi$$

(2) Cylindrical to spherical.

$$\rho = \sqrt{r^2 + z^2}, \theta = \theta, \varphi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$

11.7.5. **Homework Set #7**

- Read 11.7 (pages 822-826).
- Do exercises on pages 827-828:
  1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 99, 101, 121-124