Lecture 6

11.6. Surfaces in Space

Goals: (1) Recognize and write equations for cylindrical surfaces.

(2) Recognize and write equations for quadric surfaces.

(3) Recognize and write equations for surfaces of revolution.

Questions: We have learned two special types of surfaces…

- What is the sphere formula? (11.2)
- What is the general form of equation for a plane? (11.5)
- What are the traces on the coordinate planes?

11.6.1. Cylindrical surfaces (Cylinders)

(1) Definition:

- Let C be a curve in a plane, and let L be a line not in a parallel plane. The set of all lines parallel to L and passing through a point at C is called a ***cylinder***. C is called the ***generating curve*** (or ***directrix***) of the cylinder, and the parallel lines and L are called ***rules***.
  Note: In this section, we only discuss regular cylinders, meaning that L is perpendicular to the plane containing C. Without loss of generality, we can assume that C lies on a coordinate plane.

(2) Equations:

- If L is parallel to x-axis (which implies that C lies on yz-plane), then the equation of the cylinder contains only variables y and z.
- Similarly, we can obtain the other two… What are they?

(3) Example 1: Sketching a cylinder.

- Try exercises 7-16.

11.6.2. Quadric surfaces

An equation of a quadric surface is a quadratic equation with variables x, y and z: $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0$. We can always get a simplified form by completing the squares appropriately. The traces of quadric surfaces are conics (ellipse, parabola, parabola).

There are 6 basic types of quadric surfaces with their standard equations:

(1) Ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Note: A sphere is a special case of ellipsoid. a, b and c are nonzero. It must be three $+$, no $-$

(2) Hyperboloid of one sheet:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

Note: When $a=b$, it can be obtained by rotating the hyperbola around the real axis. As long as two $+$ one $-$, it will be a hyperboloid of one sheet.

(3) Hyperboloid of two sheets:

$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Note: When $a=b$, it can be obtained by rotating the hyperbola around the imaginary axis. As long as one $+$ two $-$, it will be a hyperboloid of two sheets.

(4) Elliptic cone:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

Note: Similar to hyperboloid, but the right-hand side is zero.

(5) Elliptic paraboloid:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$$

Note: When $a=b$, it can be obtained by rotating a parabola around the symmetric axis. No z^2 term, but just z.

(6) Hyperbolic paraboloid:

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$$

or

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-z=0$$

Note: No rotation involved in this case. Sometimes we call this surface a saddle.

(7) Examples 2, 3: Sketching a quadric surface. Need some algebra work.

- Try exercises 1-6, 19-30.

(8) Example 4: Sketching a quadric surface not centered at origin. Need more algebra work: to complete the square.

- Try exercises 31-32.

11.6.3. Surfaces of revolution

(1) Given the radius function $y=r(z)$ in the yz-plane. When we revolve it about the z-axis, the equation will be: $x^2+y^2=[r(z)]^2$.
Note: We need to solve for y is it's not in this form.

(2) Given the radius function $z=r(x)$ in the zx-plane. When we revolve it about the x-axis, the equation will be: $y^2+z^2=[r(x)]^2$.

(3) Given the radius function $x=r(y)$ in the xy-plane. When we revolve it about the y-axis, the equation will be: $z^2+x^2=[r(y)]^2$.

(4) Example 5: Finding an equation for a surface of revolution.

- Try exercises 47-52.

(5) Example 6: Finding a generating curve for a surface of revolution.

- Try exercises 53-54.

11.6.4. Homework Set #6

- Read 11.6 (pages 812-819).
- Do exercises on pages 820-821:
  1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27, 29, 31, 43, 45, 47, 49, 51, 69-72