Lecture 2

11.2. Space Coordinates and Vectors in Space

Goals: (1) Understand the three-dimensional rectangular coordinate system.

(2) Analyze vectors in space.

(3) Use three-dimensional vectors to solve real-life problems.

Questions:

- What is two-dimensional coordinate system? What are quadrants?
- What is midpoint rule?
- What is the equation of a circle?

11.2.1. Coordinates in space

(1) Definitions

- Axes: x-axis, y-axis, z-axis.
- Origin: $(0,0,0)$.
- Coordinate planes: xy-plane, yz-plane, zx-plane.
- Octants: eight octants.
- Ordered triple: (x,y,z) is a point in space with three coordinates.
- Try exercises 1-24

(2) Distance formula in space

- The distance between (x_1,y_1,z_1) and (x_2,y_2,z_2) is
  $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
- Example 1 (page 774)
- Try exercises 25-34

(3) Midpoint rule in space

- The midpoint of the line segment joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is
  $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$.
- Try exercises 35, 36

(4) Standard equation of sphere

- The standard equation of sphere with center at (x_0,y_0,z_0) and radius r is
  $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$.
- Example 2 (page 774)
- Try exercises 37-44
  Equations define surfaces!
- Try exercises 45-48
  Inequalities define solids!

11.2.2. Vectors in space

(1) Definition:

- In space, a vector is denoted by ordered triple (component form): $\vec{v} = \langle v_1, v_2, v_3 \rangle$.
- Zero vector is denoted by $\vec{0} = \langle 0, 0, 0 \rangle$.
- Standard unit vectors are: $\vec{\imath} = \langle 1, 0, 0 \rangle$, $\vec{\jmath} = \langle 0, 1, 0 \rangle$, and $\vec{k} = \langle 0, 0, 1 \rangle$. See Figure 11.19.
- Vector can be represented by standard unit vectors: $\vec{v} = \langle v_1, v_2, v_3 \rangle = v_1\vec{\imath} + v_2\vec{\jmath} + v_3\vec{k}$.
- Two vectors (nonzero) $\vec{u}, \vec{v}$ are ***parallel*** if there is some scalar c such that $\vec{u} = c\vec{v}$.

(2) Questions:

- Question: How do I know if two vectors are equal?
  Answer: Two vectors are equal if and only if their components equal to each other, respectively.
- Question: How to find component form of a vector from two given points?
  Answer: If $\vec{v}$ is a vector with initial point $P(p_1, p_2, p_3)$ and terminal point $Q(q_1, q_2, q_3)$, then the component form is:
  $\vec{v} = \overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$.
- Question: How to find norm (magnitude, length) of a vector?
  $\|\vec{v}\| = \|\langle v_1, v_2, v_3 \rangle\| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$.
- Question: How to find the unit vector in the direction of $\vec{v}$ if $\vec{v}$ is nonzero?
  Answer: Divide by $\|\vec{v}\|$. That is, $\frac{\vec{v}}{\|\vec{v}\|}$, or $\frac{1}{\|\vec{v}\|}\vec{v}$.
  Note: This is similar to vector in plane!
- Question: How to add two vectors?
  Answer: $\vec{u} + \vec{v} = \langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$.
- What is scalar multiplication?
  Answer: $c\vec{v} = c\langle v_1, v_2, v_3 \rangle = \langle cv_1, cv_2, cv_3 \rangle$.
  Note: Use your own word to describe what it means!

(3) Example 3 (page 777)
Finding component form, magnitude, unit vector, etc.

- Try exercises 53-56.

(4) Example 4 (page 778)
Determining which vector is parallel to a given one.

- Try exercises 69-72.

(5) Example 5 (page 778)
Determining whether points are collinear (on the same line).

- Try exercises 73-76.

11.2.3. Applications

(1) Application of $\vec{\imath}, \vec{\jmath}, \vec{k}$

- Using the standard unit vector notation (Example 6)
- Try exercises 59-60.

(2) Application of the formula: $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

- Measuring force using sum (Example 7)
  Try exercises 113-114.

11.2.4. Homework Set #2

- Read 11.2 (pages 775-779).
- Do exercises on pages 780-782:
  1, 3, 5, 7, 9, 11, 12, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 34, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 69, 71, 73, 75, 83, 85, 87, 97, 99, 101, 113