Lecture 1

11.1. Vectors in the Plane

Goals: (1) Write the component form of a vector.

(2) Perform vector operations and interpret the results geometrically.

(3) Write the vector as a linear combination of standard unit vectors.

(4) Use vectors to solve problems involving force or velocity.

Questions:

- What is Distance Formula?
- What is Slope Formula?
- What do you remember from Calculus I & II?

11.1.1. Component form of a vector

(1) Scalar quantities vs. Directed line segments

- Scalar quantities involve in only magnitude:
  area, volume, temperature, mass, time,…
- Directed line segments involve in both magnitude and direction:
  velocity, acceleration, force,…

(2) More about Directed Line Segments (DLS)

- Initial point P, terminal point Q. This DLS is denoted by $\overrightarrow{PQ}$.
- Two DLS are *equivalent* if they have the same *length* and *direction*.
- The length (or magnitude) is denoted by $\|\overrightarrow{PQ}\|$, also called the ***norm***.
  Note: Length, Magnitude, and Norm are used interchangeably.

(3) Vector (in the plane) is used to represent the set of all equivalent DLS.

- All equivalent DLS to $\overrightarrow{PQ}$ will still be denoted by $\overrightarrow{PQ}$. Therefore, $\overrightarrow{PQ}$ is the notation for a ***vector***. So usually vector and DLS are used interchangeably.
- Another notation for a vector is lowercase, boldface letter: $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$
- The third notation for a vector is lowercase letter with arrow above it:
  $\vec{u}, \vec{v}, \vec{w}$ and so on. The norms are $\|\vec{u}\|$, $\|\vec{v}\|$, $\|\vec{w}\|$.
- The first and third notations are our preferred ones! For example, $\vec{v} = \overrightarrow{PQ}$

(4) Example 1 (page 764)

- How to find norms of those vectors?
  Use Distance Formula!
- How to determine their direction?
  Use Slope Formula!
- Question: What does this example tell us about?
  Answer: every vector can be chosen with initial point $(0, 0)$.

(5) Component form of $\vec{v} = \overrightarrow{PQ}$

- If $P = (0, 0)$, $Q = (v_1, v_2)$, then $\vec{v} = \langle v_1, v_2 \rangle$ denotes the component form of $\vec{v}$.
- In particular, $\langle 0, 0 \rangle$ is called a zero vector, denoted by $\vec{0}$.
  That is, $\vec{0} = \langle 0, 0 \rangle$.
- Fact: two vectors $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ are ***equal*** if and only if $u_1 = v_1$ and $u_2 = v_2$. Therefore, a vector can be uniquely denoted by its component form!
- How to convert DLS form to Component form?
  Given $P\ (p_1, p_2)$ and $Q\ (q_1, q_2)$ as initial and terminal points of $\overrightarrow{PQ}$. Then the component form is $\langle q_1 - p_1, q_2 - p_2 \rangle$. How do you find the norm of the vector?
- How to convert Component form to DLS form?
  Given $\vec{v} = \langle v_1, v_2 \rangle$. Then we can take $P\ (0, 0)$ and $Q\ (v_1, v_2)$. This is said to be in standard position.
- $\vec{v} = \langle v_1, v_2 \rangle$ is called a ***unit vector***, if the norm is 1, that is, $\|\vec{v}\| = 1$.
- Note: $\|\vec{v}\| = 0$ if and only if $\vec{v}$ is the zero vector $\vec{0}$.

(6) Example 2 (page 765)

- Try exercises 1-16.

11.1.2. Vector operations

(1) Definition: Assume that two vectors are given in component form:
$\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, and c is a scalar (number).

- Scalar Multiple: $c\vec{v} = c\langle v_1, v_2 \rangle = \langle cv_1, cv_2 \rangle$
  Note: if c is positive, then $c\vec{v}$ is the vector c times as long as $\vec{v}$, with same direction. What if c is negative?
- Sum: $\vec{u} + \vec{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$
  See Figure 11.7 or 11.8 (page 764)
- Negative: $-\vec{v} = (-1)\vec{v} = \langle -v_1, -v_2 \rangle$
  Same norm with opposite direction.
- Difference: $\vec{u} - \vec{v} = \vec{u} + (-\vec{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$
  See Figure 11.8 (page 766)

(2) Example 3 (page 767)

- Try exercises 17-28.

(3) Properties of vector operations: Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be vectors, c, d be scalars.

- Additive commutativity: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- Additive associativity: $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
- Additive identity: $\vec{u} + \vec{0} = \vec{u}$
- Additive inverse: $\vec{u} + (-\vec{u}) = \vec{0}$
- Multiplicative asssociativity: $c(d\vec{u}) = (cd)\vec{u}$
- Unit scalar: $1\vec{u} = \vec{u}$
- Zero scalar: $0\vec{u} = \vec{0}$
- Distributivity: $(c + d)\vec{u} = c\vec{u} + d\vec{u}$; $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

Can you prove them using component form?

(4) Scalar multiple property: Let $\vec{v}$ be a vector, c be a scalar.
- $\|c\vec{v}\| = |c| \cdot \|\vec{v}\|$

(5) Normalization of a vector
- Question: How to find the unit vector in the direction of $\vec{v}$ if $\vec{v}$ is nonzero?
  Answer: Divide by $\|\vec{v}\|$. That is, $\frac{\vec{v}}{\|\vec{v}\|}$, or $\frac{1}{\|\vec{v}\|}\vec{v}$.

(6) Example 4 (page 769)
- Try exercises 37-40.

(7) Note: Usually $\|\vec{u}+\vec{v}\| \neq \|\vec{u}\| + \|\vec{v}\|$. In general, $\|\vec{u}+\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$. This is known as "triangle inequality".

11.1.3. Standard unit vectors

(1) Definition
- Two special vectors $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$ are called ***standard unit vectors*** in the plane. They are denoted by: $\vec{\imath} = \langle 1, 0\rangle$, and $\vec{\jmath} = \langle 0, 1\rangle$.

(2) Any vector can be represented as a linear combination of $\vec{\imath}$ and $\vec{\jmath}$.
- $\langle v_1, v_2\rangle = v_1\langle 1, 0\rangle + v_2\langle 0, 1\rangle = v_1\vec{\imath} + v_2\vec{\jmath}$.

(3) Example 5 (page 769)
- Try exercises 31-36.

(4) A unit vector can be written in the form:
$$\vec{u} = \langle \cos\theta, \sin\theta\rangle = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$$
Therefore, any vector $\vec{v}$ can be written in the form:
$$\vec{v} = \|\vec{v}\|\langle \cos\theta, \sin\theta\rangle = \|\vec{v}\|\cos\theta\,\vec{\imath} + \|\vec{v}\|\sin\theta\,\vec{\jmath}$$
See Figure 11.11 (page 768).

(5) Example 6 (page 770)
- Try exercises 51-58.

11.1.4. Applications

Use the form: $\vec{v} = \|\vec{v}\|\langle \cos\theta, \sin\theta\rangle = \|\vec{v}\|\cos\theta\,\vec{\imath} + \|\vec{v}\|\sin\theta\,\vec{\jmath}$
- Finding the resultant force (Example 7)
- Finding a velocity (Example 8)

11.1.5. Homework Set #1

- Read 11.1 (pages 764-771).
- Do exercises on pages 771-774:
  1, 3, 5, 7, 9, 15, 17, 19, 21, 23, 27, 29, 31, 33, 35, 37, 39, 41, 45, 47, 51, 53, 55, 57, 63, 65, 69, 71, 75, 79, 81, 83, 87, 93, 95-100