<u>Trigonometric Identities</u>

Type I (memorize)

Reciprocal Identities:

$\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Negative-Angle Identities:

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$ $\sec(-\theta) = \sec\theta$ $\csc(-\theta) = -\csc\theta$

Cofunction Identities:

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \csc\theta$

$\csc(90° - \theta) = \sec\theta$

Sum and Difference Identities:

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Double-Angle Identities:

$\sin 2A = 2\sin A\cos A$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases}$$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Type II (recognize and be able to use)

Half-Angle Identities:

$\cos\frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}}$

$\sin\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}}$

$\tan\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

$\tan\frac{A}{2} = \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$

Product-to-Sum Identities:

$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$

$\sin A\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

Sum-to-Product Identities:

$\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)$