Mth134 – Calculus II – Final Exam Formula Sheet

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

VOLUME, ARC LENGTH, AND SURFACE AREA

- washer/disk: $V = \pi \int_a^b (R^2 - r^2)(thickness)$
- shell: $V = 2\pi \int_a^b (radius)(height)(thickness)$
- arc length: $s = \int_a^b ds$ surface area: $S = 2\pi \int_a^b (radius) ds$

$ds = \sqrt{1 + [f'(x)]^2}\,dx$	if in terms of x
$ds = \sqrt{1 + [g'(y)]^2}\,dy$	if in terms of y
$ds = \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt$	parametric

WORK AND FLUID FORCE

- $W = FD$ Work = Force × Distance
- $F = ma$ Force = mass × acceleration
- $F = kd$ Hooke's Law for springs
- $W = \int_a^b F(x)dx$
- $P = wh$ Pressure = weight-density × depth
- $F = PA$ Fluid force = pressure × area

SOME TRIGONOMETRIC IDENTITIES

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

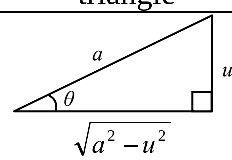
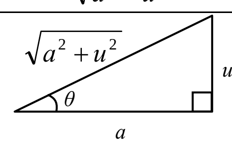
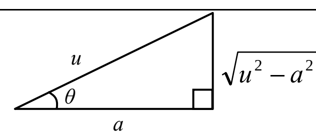
$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

CENTER OF MASS

$$\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$$

point masses	*planar region*	
$M_y = \sum_{i=1}^{n} m_i x_i$	$M_y = \rho \int_a^b x\left[f(x) - g(x)\right]dx$	moment about the *y*-axis
$M_x = \sum_{i=1}^{n} m_i y_i$	$M_x = \rho \int_a^b \left[\frac{f(x)+g(x)}{2}\right]\left[f(x) - g(x)\right]dx$	moment about the *x*-axis
$m = \sum_{i=1}^{n} m_i$	$m = \rho \int_a^b \left[f(x) - g(x)\right]dx$	total mass of the system

TRIGONOMETRIC SUBSTITUTION

form	substitution	triangle
$\sqrt{a^2 - u^2}$	$u = a\sin\theta$	
$\sqrt{a^2 + u^2}$	$u = a\tan\theta$	
$\sqrt{u^2 - a^2}$	$u = a\sec\theta$	

POLAR COORDINATES

$$x = r\cos\theta \qquad y = r\sin\theta \qquad \tan\theta = \frac{y}{x} \qquad r^2 = x^2 + y^2$$

$$A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$$

POWER SERIES, TAYLOR, MACLAURIN, ETC

***n*th Taylor polynomial for *f* at c**: $P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$

n*th Maclaurin polynomial for *f: $P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \cdots + \frac{f^{(n)}(0)}{n!}(x)^n$

power series centered at *c*: $\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots + a_n(x-c)^n + \cdots$

Taylor series for *f* at c: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$ (When $c = 0$, it is the **Maclaurin series**.)

Power Series for Elementary Functions

Function	*Interval of Convergence*
$\frac{1}{x} = 1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-\cdots(-1)^n(x-1)^n+\cdots$	$0 < x < 2$
$\frac{1}{1+x} = 1-x+x^2-x^3+x^4-\cdots+(-1)^n x^n+\cdots$	$-1 < x < 1$
$\ln x = (x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\cdots+\frac{(-1)^n(x-1)^n}{n}+\cdots$	$0 < x \le 2$
$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots+\frac{x^n}{n!}+\cdots$	$-\infty < x < \infty$
$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$	$-\infty < x < \infty$
$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$	$-\infty < x < \infty$
$\arctan x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\cdots+\frac{(-1)^n x^{2n+1}}{2n+1}+\cdots$	$-1 \le x \le 1$
$\arcsin x = x+\frac{x^3}{2\cdot 3}+\frac{1\cdot 3x^5}{2\cdot 4\cdot 5}-\frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7}+\cdots+\frac{(2n)!\,x^{2n+1}}{(2^n n!)^2(2n+1)}+\cdots$	$-1 \le x \le 1$
$(1+x)^k = 1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}+\frac{k(k-1)(k-2)(k-3)x^4}{4!}+\cdots$	$-1 < x < 1$*

* *The convergence at* $x = \pm 1$ *depends on the value of* k.