<u>Exam 3 Review</u>

Note: This is not a complete list of topics – you should study your lecture notes and homework in addition to reviewing the items listed here.

1. basic integration rules (§ 8.1)
   a. be sure to memorize all the basic integration rules as before
   b. strategies (pg 521)

technique	*example*
expand (numerator)	$(1+e^x)^2 = 1+2e^x+e^{2x}$
separate numerator	$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
complete the square	$\frac{1}{x^2-6x+13} = \frac{1}{(x-3)^2+4}$
divide improper rational function	$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
add and subtract terms in the numerator	$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$
use trigonometric identities	$\cot^2 x = \csc^2 x - 1$
multiply and divide by a Pythagorean conjugate	$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x}$ $= \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$

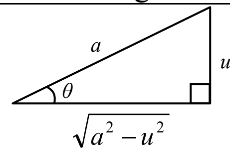
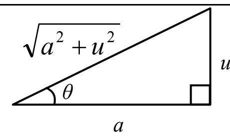
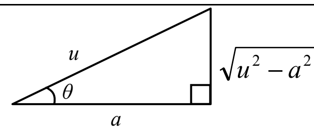
2. integration by parts (§ 8.2)
   a. $\int u\,dv = uv - \int v\,du$
   b. tabular method:

ex $\int x^2 e^{3x}\,dx$

u		dv
x^2	$+$	e^{3x}
$2x$	$-$	$\frac{1}{3}e^{3x}$
2	$+$	$\frac{1}{9}e^{3x}$
0		$\frac{1}{27}e^{3x}$

$$\int x^2 e^{3x}\,dx = \tfrac{1}{3}x^2e^{3x} - \tfrac{2}{9}xe^{3x} + \tfrac{2}{27}e^{3x} + C$$

3. trigonometric substitution (§ 8.4)

form	substitution	triangle
$\sqrt{a^2 - u^2}$	$u = a\sin\theta$	
$\sqrt{a^2 + u^2}$	$u = a\tan\theta$	
$\sqrt{u^2 - a^2}$	$u = a\sec\theta$	

4. partial fractions (§ 8.5)
   a. if the degree of the numerator is more than the degree of the denominator, divide first
   b. factor the denominator completely
   c. linear factors:
   $$\frac{(\quad)}{(px+q)^n(\quad)} \Rightarrow \frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_n}{(px+q)^n}$$
   d. quadratic factors:
   $$\frac{(\quad)}{(ax^2+bx+c)^n(\quad)} \Rightarrow \frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

5. integration by tables (§ 8.6)
   be careful when substituting for *u* – when in doubt, do the complete substitution

6. L'Hospital's Rule (§ 8.7)
   a. for indeterminate forms $0/0$ or ∞/∞, $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$
   b. for the indeterminate form $0 \cdot \infty$, rewrite the limit to fit the form $0/0$ or ∞/∞
   c. for indeterminate forms 0^0 and 1^∞, let $y = \lim f(x)$, take the natural log of both sides, apply L'Hospital's rule to find $\ln y$, and then the solution is $e^{\ln y}$.

7. improper integrals (§ 8.8)
   a. $\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$ (similarly for $\int_{-\infty}^a f(x)dx$ and $\int_{-\infty}^\infty f(x)dx$)
   b. If *f* has an infinite discontinuity at *b* (i.e. an asymptote), then
   $\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx$ (similarly if the discontinuity is at *a* or in between *a* and *b*)