<u>**Exam 1 Review**</u>

Note: This is not a complete list of topics – you should study your lecture notes and homework in addition to reviewing the items listed here.

1. natural logarithmic function
   a. definition: $\ln x = \int_1^x \frac{1}{t}\,dt$
   b. properties:
      - $\ln 1 = 0$
      - $\ln e = 1$
      - $\ln(ab) = \ln a + \ln b$
      - $\ln(a^n) = n \ln a$
      - $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
   c. differentiation:
      $$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$
   d. logarithmic differentiation
      i. take the natural log of both sides of the equation
      ii. use the properties of natural log to simplify the equation
      iii. differentiate both sides of the equation with respect to x
      iv. solve for y'
      v. substitute back in for y, so your answer is in terms of x only
   e. integration:
      $$\int \frac{1}{u}\,du = \ln|u| + C$$

2. inverse functions
   a. A function has an inverse if and only if it is one-to-one.
   b. You should know the strategy for finding an inverse.
   c. If f and g are inverses, $g'(x) = \dfrac{1}{f'(g(x))}$, $f'(g(x)) \neq 0$.

3. exponential functions
   a. $f(x) = \ln x$ and $g(x) = e^x$ are inverses
   b. $\frac{d}{dx}(e^u) = e^u u'$
   c. $\int e^u\,du = e^u + C$

4. bases other than e
   a. definition:
      i. $a^x = e^{(\ln a)x}$
      ii. $\log_a x = \frac{1}{\ln a} x$
   b. $f(x) = a^x$ and $g(x) = \log_a x$ are inverses
   c. derivatives:
      i. $\frac{d}{dx}\left(a^u\right) = (\ln a) a^u u'$
      ii. $\frac{d}{dx}\left(\log_a u\right) = \frac{1}{(\ln a)u} u'$
   d. integrals: $\int a^u \, du = \frac{1}{\ln a} a^u + C$

5. inverse trigonometric functions
   a. definitions (see handout)
      i. know how to evaluate an inverse trigonometric function
      ii. know how to express in terms of x – for example: $\cos(\arcsin 2x)$
   b. derivatives

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[arc\cot u] = \frac{-u'}{1+u^2} \qquad \frac{d}{dx}[arc\sec u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[arc\csc u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

   c. integrals

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$