An Application of Simpson's Rule - Solution

Problem: Suppose we want to know roughly how much water is flowing through a particular stream. Using a combination of water level (usually referred to as *stage*) and previous measurements, we collect flow data over 30 minute time intervals as shown in the table below. Use Simpson's Method to approximate the total flow (in ft^3) for the Vermillion River Near Empire, MN from 5am to 3pm on Sunday, March 9th, 2003.

Solution:

If we let $f(t)$ = flow through the river at time *t*, our goal is then to approximate $\int_{5\,am}^{3\,pm} f(t)dt$.

If we collect data every 30 minutes, Δt = 30 minutes, and we can then calculate using Simpson's Method:

$$\int_{5\,am}^{3\,pm} f(t)dt \approx \frac{\Delta t}{3}\left[f(5\,am)+4f(5:30\,am)+2f(6\,am)+\cdots+4f(2:30\,pm)+f(3\,pm)\right]$$

$$\approx \frac{30}{3}\left[2400+4*2520+2*2520+\cdots+4*2400+2520\right]$$

$$\approx 1{,}434{,}000\ ft^3$$

obtained from http://waterdata.usgs.gov/nwis/rt

date & time	ft^3/sec	ft^3/min	Simpson's Calculation	
3/9/2003 5:00	40	2400	2400*30 =	72000
3/9/2003 5:30	42	2520	**4***2520*30 =	302400
3/9/2003 6:00	42	2520	**2***2520*30 =	151200
3/9/2003 6:30	43	2580	**4***2580*30 =	309600
3/9/2003 7:00	43	2580	**2***2580*30 =	154800
3/9/2003 7:30	43	2580	**4***2580*30 =	309600
3/9/2003 8:00	43	2580	**2***2580*30 =	154800
3/9/2003 8:30	42	2520	.	302400
3/9/2003 9:00	42	2520	.	151200
3/9/2003 9:30	42	2520	.	302400
3/9/2003 10:00	39	2340		140400
3/9/2003 10:30	36	2160		259200
3/9/2003 11:00	34	2040		122400
3/9/2003 11:30	33	1980		237600
3/9/2003 12:00	33	1980	.	118800
3/9/2003 12:30	34	2040	.	244800
3/9/2003 13:00	35	2100	.	126000
3/9/2003 13:30	37	2220	**4***2220*30 =	266400
3/9/2003 14:00	38	2280	**2***2280*30 =	136800
3/9/2003 14:30	40	2400	**4***2400*30 =	288000
3/9/2003 15:00	42	2520	2520*30 =	151200
				4,302,000 * ⅓ = 1,434,000 ft^3