Prerequisite knowledge for Mth133 – Calculus I

Notation:

1. function: $y = f(x)$
2. set notation
   ex. $\{x \mid x < 4\}$, $\{1, 3, 5, 7\}$, $\{y \mid 2 \le y < 5\}$
3. interval notation
   $[a, b]$, $[a, b)$, $[a, \infty)$, etc.
4. $\Delta \sim$ "change in" (i.e. Δx, Δy, etc.)

Formulas:

1. Pythagorean Theorem: $a^2 + b^2 = c^2$
2. distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. circumference of a circle: $C = 2\pi r$
4. area of a circle: $A = \pi r^2$
5. area of a trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$
6. area of a sector: $A = \frac{1}{2}r^2\theta$
7. volume of a sphere: $V = \frac{4}{3}\pi r^3$
8. volume of a cylinder: $V = \pi r^2 h$
9. volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Basic mathematics:

1. exponents
   a. $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{\text{n times}}$
   b. $a^n \cdot a^m = a^{n+m}$
   c. $\dfrac{a^n}{a^m} = a^{n-m}$
   d. $(a^n)^m = a^{nm}$
   e. $(a^n b^m)^p = a^{np} b^{mp}$
   f. $\left(\dfrac{a^n}{b^m}\right)^p = \dfrac{a^{np}}{b^{mp}}$
   g. $a^{\frac{1}{n}} = \sqrt[n]{a}$
   h. $a^{-n} = \dfrac{1}{a^n}$
   i. $a^{\frac{m}{b}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

2. simplifying square roots
   ex. $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$
   ex. $\sqrt{9+16} \ne \sqrt{9} + \sqrt{16} = 3 + 4 = 7$, $\sqrt{9+16} = \sqrt{25} = 5$

Algebra:

1. domain and range of functions (see pg 21)
2. function composition (see pg 25) – $(f \circ g)(x) = f(g(x))$
3. function symmetry (see pg 5 and pg 26)
   a. $y = f(x)$ is *even* if $f(-x) = f(x)$. (*Even* symmetry is symmetry over the *y*-axis.)
   b. $y = f(x)$ is *odd* if $f(-x) = -f(x)$. (*Odd* symmetry is symmetry with respect to the origin.)
4. function transformation (see pg 23)
   a. $y = f(x-c)$ shifts the graph of $y = f(x)$ *c* units **right**
   b. $y = f(x+c)$ shifts the graph of $y = f(x)$ *c* units **left**
   c. $y = f(x)-c$ shifts the graph of $y = f(x)$ *c* units **downward**
   d. $y = f(x)+c$ shifts the graph of $y = f(x)$ *c* units **upward**
   e. $y = -f(x)$ reflects the graph of $y = f(x)$ over the *x*-axis
   f. $y = f(-x)$ reflects the graph of $y = f(x)$ over the *y*-axis
   g. $y = k\ f(x)$ stretches the graph of $y = f(x)$ **vertically** by a factor of *k*.
   h. $y = f(kx)$ compresses the graph of $y = f(x)$ **horizontally** by a factor of $\frac{1}{k}$.
5. piecewise-defined functions (see ex. 3, pg 21)

   ex. $f(x) = \begin{cases} 2x-1 & , x \le 1 \\ x^2 & , x > 1 \end{cases}$
6. lines (see section P2, pg 10)
   a. slope: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
   b. General form: $Ax + By + C = 0$, (*A* and *B* not both zero)
   c. Vertical line: $x = a$
   d. Horizontal line: $y = b$
   e. Point-slope form: $y - y_1 = m(x - x_1)$
   f. Slope-intercept form: $y = mx + b$
   g. parallel lines: $m_1 = m_2$
   h. perpendicular lines: $m_1 = -\dfrac{1}{m_2}$
7. solving equations
   a. linear
   b. quadratic
      i. using factoring:

      ex. $x^2 + 5x + 6 = 0 \quad \Rightarrow \quad (x+3)(x+2) = 0$

      $\Rightarrow \quad x+3 = 0 \ \ or \ \ x+2 = 0 \quad \Rightarrow \quad x = -3 \ or \ x = -2$

      solution set: {-3, -2}

      ii. quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. factoring

   ex. $x^2 - 5x + 6 = (x-3)(x-2)$

   $a^2 - b^2 = (a+b)(a-b)$

   $a^2 + 2ab + b^2 = (a+b)^2$

   $a^2 - 2ab + b^2 = (a-b)^2$

   etc…

9. inequalities
   a. linear

      ex. $3x + 7 \le 13 \;\Rightarrow\; 3x \le 6 \;\Rightarrow\; x \le 2 \;\; or \;\; (-\infty, 2]$

   b. compound

      ex. $-1 \le 2x + 3 \le 5 \;\Rightarrow\; -4 \le 2x \le 2 \;\Rightarrow\; -2 \le x \le 1 \;\; or \;\; [-2, 1]$

10. simplifying complex fractions (multiply the numerator and denominator by the LCD)

    ex. $\dfrac{\frac{1}{x}+2}{\frac{2}{y}-\frac{1}{x}} = \dfrac{\left(\frac{1}{x}+2\right)}{\left(\frac{2}{y}-\frac{1}{x}\right)} \cdot \dfrac{xy}{xy} = \dfrac{y+2xy}{2x-y}$

11. simplifying rational expressions (factor and reduce)

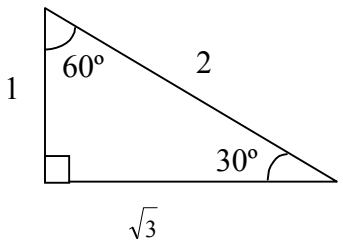
    ex. $\dfrac{x^2-5x+6}{x^2-x-2} = \dfrac{(x-3)(x-2)}{(x+1)(x-2)} = \dfrac{x-3}{x+1},\; x \ne 2$

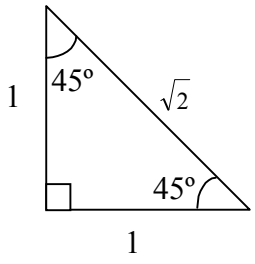
12. rationalizing the denominator (multiply the numerator and denominator by the conjugate)

    ex. $\dfrac{1}{2+\sqrt{3}} = \dfrac{1}{\left(2+\sqrt{3}\right)} \cdot \dfrac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)} = \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

Trigonometry

1. SOH-CAH-TOA: $\text{sine} = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\text{cosine} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\text{tangent} = \dfrac{\text{opposite}}{\text{adjacent}}$
2. π radians = 180º, $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, $\frac{\pi}{3} = 60°$, $\frac{\pi}{2} = 90°$, etc.
3. know the *unit circle* – visit the course website or the front page of your text.
4. 30-60-90 and 45-45-90 triangles:





Note: This short summary does **not** include all of the tools necessary for success in Mth133. You should be prepared to review certain topics throughout the course when necessary.