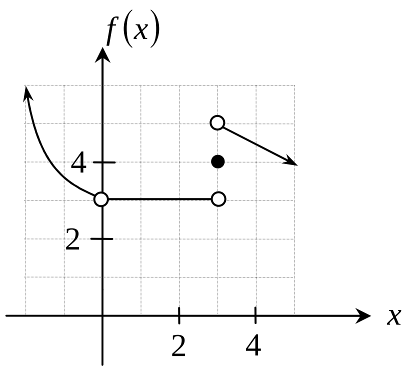
Mth133 – Calculus – **Practice Exam Questions**

NOTE: These questions should not be taken as a complete list of possible problems. They are merely intended to be examples of the difficulty level of the regular exam questions. Questions are listed by section, so be sure to only study the questions from the sections your particular exam is covering.

This is only a *list* of questions – use a separate sheet to work out the problems.

1. (1.2 and 1.4) Use the given graph to answer each question.



a. $\lim_{x\to 3^-} f(x)$

b. $\lim_{x\to 3^+} f(x)$

c. $\lim_{x\to 0} f(x)$

d. $f(0)$

e. Does the limit of $f(x)$ exist as $x \to 3$? Why or why not?

f. Is the graph of *f* continuous at $x = 0$? Why or why not?

2. Find the following limits, if they exist. If the limit is an infinite limit, specify ∞ or $-\infty$. Making a table of values is **not** an acceptable method.

a. (1.3) $\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3}$

b. (1.3) $\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x - 3}$

c. (1.3) $\lim_{x\to 3} \sqrt{x+1}$

d. (1.4) $\lim_{x\to 5^-} \frac{8}{5 - x}$

e. (1.4) $\lim_{x\to 3^+} \frac{[\![3 - x]\!]}{4 - x}$

3. (1.2) Complete the table and use the result to estimate the limit.

$\lim_{x\to 4} \frac{\frac{x}{x+1} - \frac{4}{5}}{x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

4. (1.2) Find the limit L. Then find a $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$. To receive full credit, you must show that your δ makes the above statement true.

$$\lim_{x \to 2}(4x - 3)$$

5. (1.3) Find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ if $f(x) = 3x + 1$.

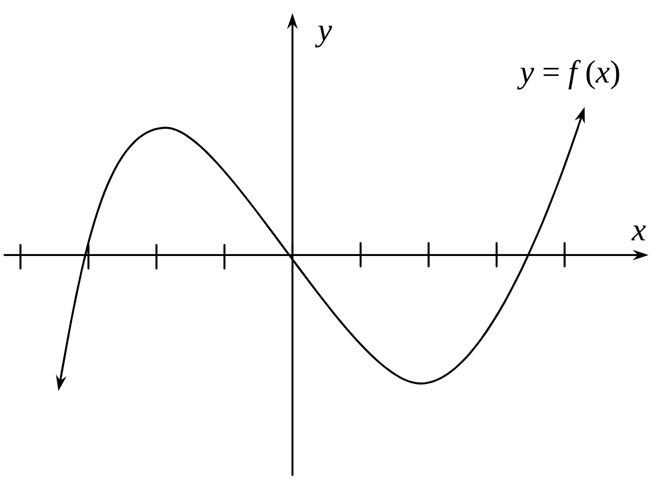
6. (1.4) Use the Intermediate Value Theorem to show that there is a root to the equation $x^3 + x - 1 = 0$ in the interval $(0,1)$.

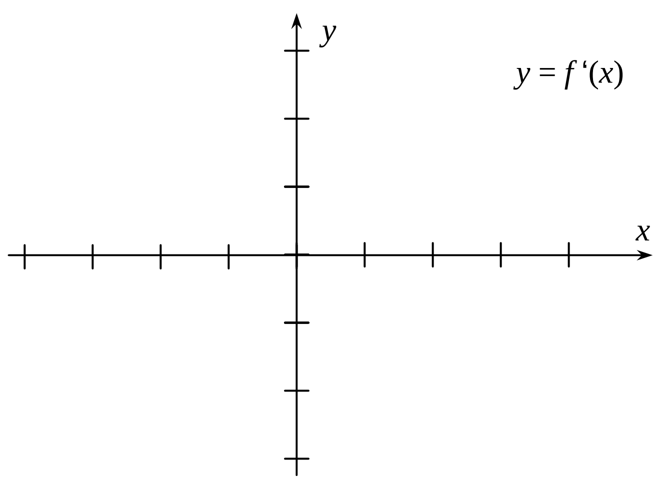
7. (1.4) Let $f(x) = [\![x]\!]$. (the greatest integer function of x) Note that $f(0) = 0$ and $f(1) = 1$ but there is no x so that $f(x) = \frac{1}{2}$. Why doesn't the Intermediate Value Theorem apply to f?

8. (1.5) Find all vertical asymptotes of the function $f(x) = \dfrac{x^2 - 4}{x^2 + 7x + 10}$.

9. (2.1) Using the definition of the derivative, find $f'(x)$ if $f(x) = \dfrac{1}{x}$.

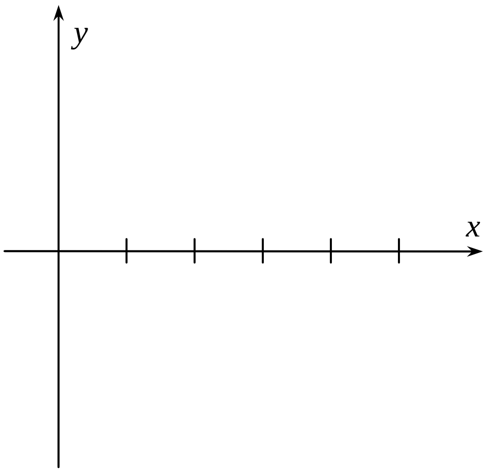
10. (2.1) Given the graph of $f(x)$ below, sketch the graph $f'(x)$.

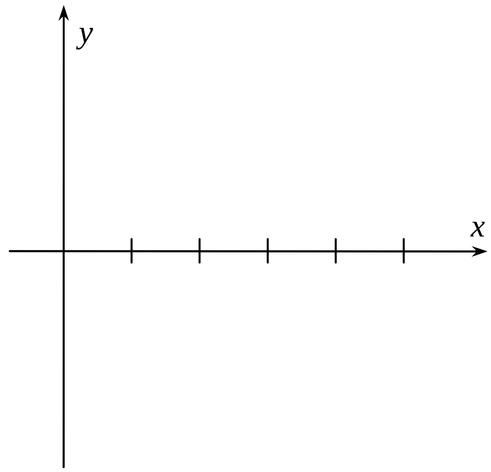




11. (2.1) If an arrow is shot upward on the moon with a velocity of 58 m/s, its height (in meters) after t seconds is given by $H = 58t - 0.83t^2$. At what time will the arrow reach its maximum height?

12. (2.1) Draw two different continuous functions whose derivatives do not exist at $x = 2$.





13. Find the derivative. DO NOT SIMPLIFY YOUR ANSWER.

a. (2.2) $y = x^{\pi}$

b. (2.4) $g(x) = (x^2 - 3)^5$

c. (2.4) $y = \sin(\cos^2 x)$

d. (2.2) $y = 3x^5 + 2x^3 - 6x^2 + 4$

e. (2.3) $f(x) = \sec x \csc x$

f. (2.3) $y = \dfrac{\sec x}{x^2}$

14. (2.4) Find an equation for the tangent line to the curve $y = \sin 2x$ at the point $(\pi, 0)$.

15. (2.5) Find $\dfrac{dy}{dx}$ if $x^2 y + x^3 y^3 = 4$

16. (2.5) Find $\dfrac{d^2 y}{dx^2}$ if $x^2 - y^2 = 16$. (Notice that I am asking for the *second* derivative.)

17. (2.6) A 13-ft ladder resting on horizontal ground is leaning against a vertical wall when its base starts to slide away from the wall. By the time the base is 12 ft from the wall, the base is moving at the rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall then?

18. (3.1) Find any critical numbers of the function $f(x) = \dfrac{4x}{x^2 + 1}$.

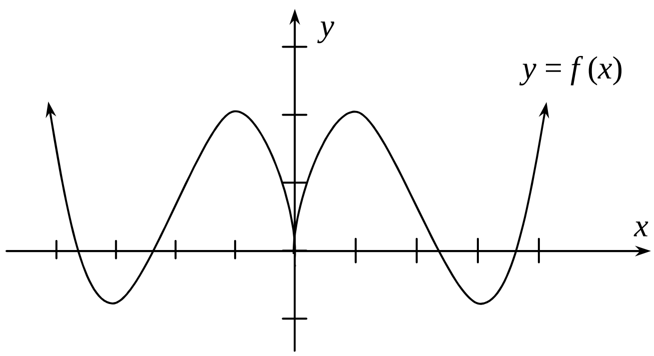
19. (3.1) Locate the absolute extrema of the function $f(x) = 2x^3 - 6x$ on the interval $[0, 2]$.

20. (3.2) State why the Mean Value Theorem can be applied to $f(x) = \sqrt{x}$ on the closed interval $[1, 9]$ and find any value(s) c in the open interval $(1, 9)$ such that $f'(c) = \dfrac{f(9) - f(1)}{9 - 1}$.

21. (3.3) Find any *x*-value(s) where $f(x) = (x^2 - 4)^{\frac{2}{3}}$ has relative extrema.

22. (3.3) Find the open interval(s) on which the function $f(x) = x^3 - 6x^2 + 15$ is increasing or decreasing.

23. (3.3 and 3.4) Given the graph of $y = f(x)$, draw first and second derivative charts. (Something like those shown in problem #9.) Use estimates for any critical numbers and inflection point coordinates.



f'

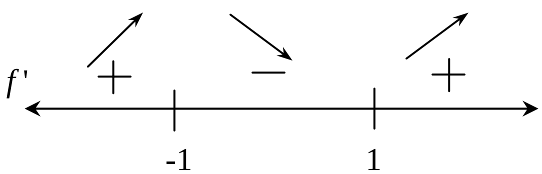
f''

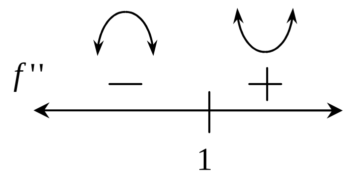
24. (3.4) Suppose you are given that $f'(2) = 0$ and that $f''(x) > 0$ for all *x*. Does $f(x)$ have a relative maximum or minimum at *x* = 2? Why?

25. (3.5) Find any horizontal asymptotes of the function $f(x)=\dfrac{3-x+2x^2}{5+2x-4x^2}$.

26. (3.5) Find $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+4x}}{4x+1}$. You must show your work to receive credit.

27. (3.6) Given the following information, sketch the graph of $y=f(x)$.

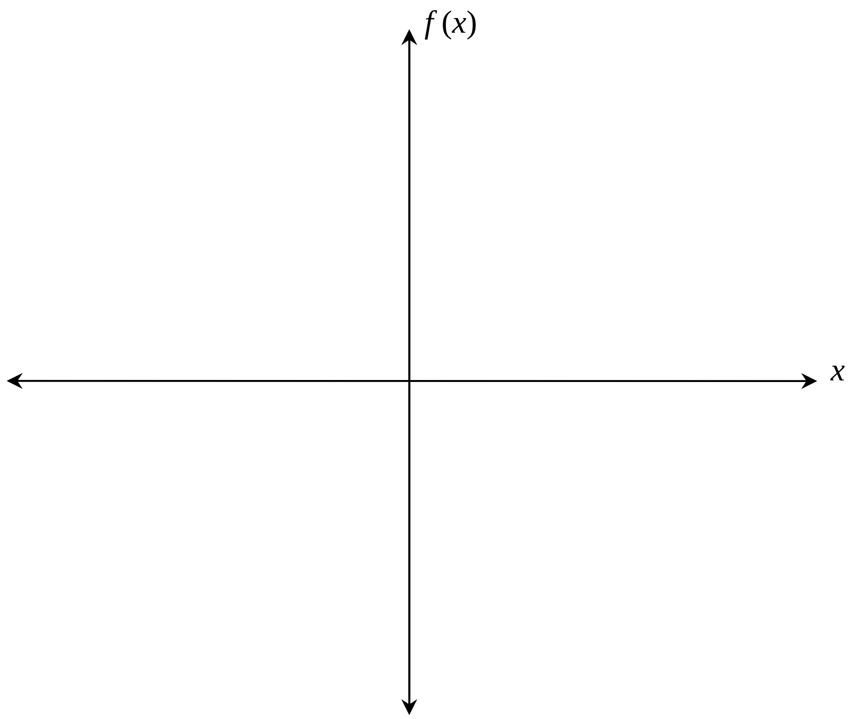




$f'(-1)=0$ $f'(1)$ does not exist

$f''(1)$ does not exist

$f(-1)=2$ $f(1)=0$



28. (3.7) Suppose you have 1350 cm^2 of material available to make a box with a square base and closed top. Find the height, width and length that would give the largest possible volume. You must use techniques learned in this course to justify your answer.

29. (3.8) Consider the equation $x^3-3x^2+3x+2=0$. Why does Newton's Method fail using an initial guess of $x_1=1$?

30. (3.8) Use Newton's Method to estimate $\sqrt{2}$ accurate to 8 decimal places.

31. (3.9) Use differentials to approximate $\sqrt[3]{26}$.

32. (3.9) Given $y = \sqrt{9 - x^2}$, find the differential dy.

33. (3.9) Suppose an oil company currently ships oil in 55-gallon cylindrical drums with radius 12 in. and height 34 in., but wants to ship in slightly narrower drums of the same height. If the company decreases the radius by $\frac{1}{4}$ in., use differentials to estimate the resulting change in the volume. (You may leave your answer in inches cubed.)

34. (4.1) Find the particular solution to the differential equation $\frac{dy}{dx} = -\frac{1}{x^2}$ given that the graph of y passes through the point $(1, 3)$.

35. Find each indefinite integral.

   a. (4.1) $\int \frac{2}{x^3}\,dx$

   b. (4.1) $\int \frac{x+1}{\sqrt{x}}\,dx$

   c. (4.1) $\int (t^2 - \sin t)\,dt$

36. (4.2) Write the following sum using summation notation. You do not need to find the sum.

$$\frac{5}{1+1} + \frac{5}{1+2} + \cdots + \frac{5}{1+10} =$$

37. (4.2) Find a formula for this sum in terms of n.

$$\sum_{i=1}^{n} \frac{(i+1)^2}{n^3}$$

38. (4.2) Estimate the area of the region between the curve $y = 4 - x^2$ and the x-axis over the interval $[1, 2]$. Use four (4) rectangles and right endpoints.

39. (4.2) Use the limit process to find the area of the region between the graph of the function $y = x^2 + 1$ and the x-axis over the interval $[0, 3]$.

40. (4.3) Evaluate the integral $\int_0^2 (x^3 + x)dx$ using a Riemann sum.

41. Find the following integrals. You must show your work to receive credit.

a. (4.1) $\int (4x^3 + 6x^2 - 1)dx$

b. (4.5) $\int \cos y \sqrt{\sin y}\, dy$

c. (4.5) $\int_{-\pi}^{\pi} \frac{x^2}{\tan x} dx$

d. (4.4 and 4.5) $\int_0^1 \frac{x^3}{(1 + x^4)^3} dx$

42. (4.6) Use the Trapezoid rule to estimate the number of square meters of land in a lot where x and y are measured in meters, as shown. The land is bounded by a stream and two straight roads that meet at right angles.

x	y
0	58
10	62
20	59
30	51
40	50
50	52
60	41
70	38
80	28
90	14
100	0