Long Division of Polynomials

1. Set up the polynomial division – leave spaces for any missing terms in the dividend.

$$3x+2\overline{\big)\; 6x^2 + 16x + 15}$$

2. Look at the first term in the divisor ($3x+2$ in this case), and determine what to multiply by to get the first term in the dividend. In this example, it is 2x, since $3x \cdot 2x = 6x^2$. Multiply 2x by the divisor and write the answer below the dividend – line up the corresponding exponents.

$$\begin{array}{rl} & \quad 2x \\ 3x+2 & \overline{\big)\; 6x^2 + 16x + 15} \\ 2x(3x+2) \rightarrow & \;\; 6x^2 + \;\; 4x \end{array}$$

3. Subtract (change the sign of your result in the previous step).

$$\begin{array}{rl} & \quad 2x \\ 3x+2 & \overline{\big)\; 6x^2 + 16x + 15} \\ & \;\; \underline{\overset{(-)}{6x^2} \overset{(-)}{+} \;\; 4x} \\ & \qquad\quad 12x \end{array}$$

4. Bring down the next term from the dividend.

$$\begin{array}{rl} & \quad 2x \\ 3x+2 & \overline{\big)\; 6x^2 + 16x + 15} \\ & \;\; \underline{\overset{(-)}{6x^2} \overset{(-)}{+} \;\; 4x} \quad \downarrow \\ & \qquad\quad 12x + 15 \end{array}$$

5. Repeat steps 2-4 as necessary.

$$\begin{array}{rl} & \quad 2x + 4 \\ 3x+2 & \overline{\big)\; 6x^2 + 16x + 15} \\ & \;\; \underline{\overset{(-)}{6x^2} \overset{(-)}{+} \;\; 4x} \\ & \qquad\quad 12x + 15 \\ 4(3x+2) \rightarrow & \qquad\quad 12x + 8 \end{array}$$

$$\begin{array}{rl} & \quad 2x + 4 \\ 3x+2 & \overline{\big)\; 6x^2 + 16x + 15} \\ & \;\; \underline{\overset{(-)}{6x^2} \overset{(-)}{+} \;\; 4x} \\ & \qquad\quad 12x + 15 \\ & \qquad\quad \underline{\overset{(-)}{12x} \overset{(-)}{+} 8} \\ & \qquad\qquad\quad 7 \end{array}$$

6. In this case, there are no further terms to drop down, so 7 is the remainder. Write the solution as the quotient on top of the division sign plus the remainder over the divisor.

$$\frac{6x^2 + 16x + 15}{3x+2} = 2x + 4 + \frac{7}{3x+2}$$

Synthetic Division of Polynomials

1. Write the dividend (the polynomial you're *dividing*) in descending powers of *x*. Then list the coefficients of each term – if a term is missing, place 0 in the appropriate position.

$$\frac{x^3+3x-7}{x+1} \Rightarrow$$

$$\begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{3} & \mathbf{-7} \end{array}$$

2. When dividing by $x - a$, place a to the left of the line in step 1. For this example, we're dividing by $x+1$, so $a=-1$.

$$\begin{array}{r|cccc} \mathbf{-1} & 1 & 0 & 3 & -7 \end{array}$$

3. Leave some space under the row of coefficients, then draw a horizontal line and bring down the first coefficient on the left.

$$\begin{array}{r|cccc} -1 & 1 & 0 & 3 & -7 \\ & \downarrow & & & \\ \hline & \mathbf{1} & & & \end{array}$$

4. Multiply a (-1 in this case) by the number brought down (1 in this case) and place the result under the next coefficient.

$$\begin{array}{r|cccc} -1 & 1 & 0 & 3 & -7 \\ & & \mathbf{-1} & & \\ \hline & 1 & & & \end{array}$$

5. Add the two numbers in the next column.

$$\begin{array}{r|cccc} -1 & 1 & 0 & 3 & -7 \\ & & -1 & & \\ \hline & 1 & \mathbf{-1} & & \end{array}$$

6. Repeat steps 4 and 5 as necessary.

$$\begin{array}{r|cccc} -1 & 1 & 0 & 3 & -7 \\ & & -1 & \mathbf{1} & \mathbf{-4} \\ \hline & 1 & -1 & \mathbf{4} & \mathbf{-11} \end{array}$$

7. In the last row, the far right number is the remainder, and working *right to left*, the others are the constant, the coefficient of *x*, the coefficient of x^2, etc.

$$\begin{array}{r|cccc} -1 & 1 & 0 & 3 & -7 \\ & & -1 & 1 & -4 \\ \hline & \mathbf{1} & \mathbf{-1} & \mathbf{4} & \mathbf{-11} \end{array}$$

$$\Rightarrow \quad x^2 - x + 4 - \frac{11}{x+1}$$