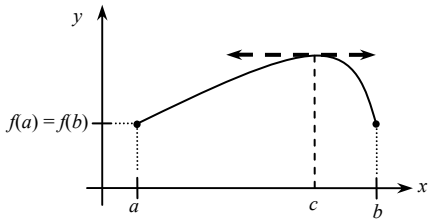
Mth133 – Calculus: Exam 3 Review

Note: This is not a complete list of topics – you should study your lecture notes and homework in addition to reviewing the items listed here.

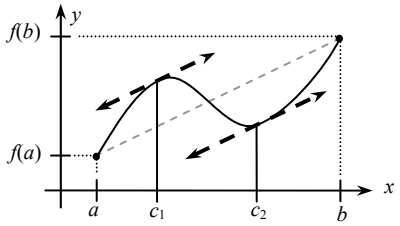
1. Critical points
   a. where $f'(x)=0$ or $f'(x)$ is undefined.

2. Testing for absolute and local max/min on a closed interval [*a,b*]:
   a. find the critical points of f
   b. find $f(a)$ and $f(b)$
   c. the largest from i.) and ii.) is the absolute max, and the smallest is the absolute min.

3. Rolle's Theorem
   a. Let f be a function that satisfies the following three hypothesis:
      i. f is continuous on the closed interval [*a,b*].
      ii. f is differentiable on the open interval (*a,b*).
      iii. $f(a)=f(b)$
   b. Then there is a number *c* in (*a,b*) such that $f'(c)=0$.



4. The Mean Value Theorem
   a. Let f be a function that satisfies the following three hypothesis:
      i. f is continuous on the closed interval [*a,b*].
      ii. f is differentiable on the open interval (*a,b*).
   b. Then there is a number *c* in (*a*,*b*) such that
   $$f'(c)=\frac{f(b)-f(a)}{b-a}$$



5. First Derivative Test

   example:

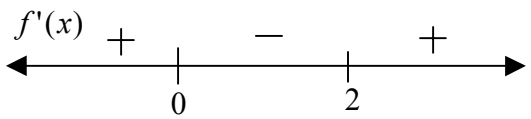
   $f(x)=5-3x^2+x^3$

   $f'(x)=-6x+3x^2$

   $f'(x)=0 \Rightarrow -6x+3x^2=0$

   $\Rightarrow -3x(2-x)=0$

   $\Rightarrow x=0 \text{ or } x=2$



   so from the first derivative test, $(0,5)$ is a rel. max, and $(2,1)$ is a rel. min.

6. Second Derivative Test

example:

$f''(x) = -6 + 6x$

$f''(0) = -6$

$f''(2) = 6$

so from the second derivative test, $(0, 5)$ is a rel. max. because f is concave down at $x = 0$, and $(2, 1)$ is a rel. min. because f is concave up at $x = 2$.

7. Limits at infinity $f(x) = \dfrac{p(x)}{q(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_2 x^2 + b_1 x + b_0}$

   a. If $n < m$, then $\lim\limits_{x \to \pm\infty} f(x) = 0$

   b. If $n = m$, then $\lim\limits_{x \to \pm\infty} f(x) = \dfrac{a_n}{b_m}$

   c. If $n > m$, then $\lim\limits_{x \to \pm\infty} f(x)$ does not exist

8. Graphing in general

   the most important parts are:

   a. intercepts
      i. y-int: set $x = 0$
      ii. x-int: set $y = f(x) = 0$
   b. asymptotes
      i. vertical: set the denominator equal to zero
      ii. horizontal: find the limits as $x \to \pm\infty$
      iii. slant: when the degree of the numerator is one more than the degree of the denominator - divide
   c. critical points (first derivative chart) – and find the actual points
   d. inflection points (second derivative chart) – again, find the points