Verifying Identities Practice – **Solutions** (My methods in italics.)

Note: There are certainly **other** ways to verify these.

1. $2-\sec^2\alpha = 1-\tan^2\alpha$

$= 1-(\sec^2\theta - 1)$ *Work with the right-hand side. (left is just as easy)*

$= 1-\sec^2\theta + 1$ *Use the identity* $\tan^2\alpha + 1 = \sec^2\alpha$

$= 2-\sec^2\theta$

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2. $\dfrac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$ *Work with the left-hand side.*

$= \dfrac{\frac{1}{\cos\theta} - 1}{1-\cos\theta}$ *Write everything in terms of sine and cosine.*

$= \dfrac{\frac{1}{\cos\theta} - 1}{1-\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta}$ *Multiply by* $\cos x$ *to clear fractions.*

$= \dfrac{1-\cos\theta}{(1-\cos\theta)\cos\theta}$ *Reduce.*

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

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3. $\sec x - \cos x = \sin x\tan x$ *Work with the left-hand side.*

$= \dfrac{1}{\cos x} - \cos x$ *Write everything in terms of sine and cosine.*

$= \dfrac{1}{\cos x} - \cos x\cdot\dfrac{\cos x}{\cos x}$ *Multiply by* $\cos x$ *to get a common denominator.*

$= \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}$

$= \dfrac{1-\cos^2 x}{\cos x}$

$= \dfrac{\sin^2 x}{\cos x}$ *Use the Pythagorean Identity.*

$= \dfrac{\sin x\sin x}{\cos x}$

$= \sin x\tan x$

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4. $\cos x + \sin x\tan x = \sec x$ *Work with the left-hand side.*

$= \cos x + \sin x\cdot\dfrac{\sin x}{\cos x}$ *Write in terms of sine and cosine.*

$= \dfrac{\cos x}{1}\cdot\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{1}\cdot\dfrac{\sin x}{\cos x}$ *Find a common denominator.*

$= \dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos x}$

$= \dfrac{1}{\cos x}$ *Pythagorean identity.*

$= \sec x$

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5. $\dfrac{\tan x + \tan y}{1-\tan x\tan y} = \dfrac{\cot x + \cot y}{\cot x\cot y - 1}$ *Work with the left-hand side. (right is just as easy)*

$= \dfrac{\frac{1}{\cot x} + \frac{1}{\cot y}}{1-\frac{1}{\cot x}\frac{1}{\cot y}}$ *Write in terms of cotangent.*

$= \dfrac{\left(\frac{1}{\cot x} + \frac{1}{\cot y}\right)}{\left(1-\frac{1}{\cot x\cot y}\right)}\cdot\dfrac{\cot x\cot y}{\cot x\cot y}$ *Multiply by* $\cot x\cot y$ *to clear fractions*

$= \dfrac{\cot y + \cot x}{\cot x\cot y - 1}$

$= \dfrac{\cot x + \cot y}{\cot x\cot y - 1}$

6. $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$

$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$

$= \cos^2 x - \sin^2 y \cos^2 x - \sin^2 y + \sin^2 y \cos^2 x$

$= \cos^2 x - \sin^2 y$

Work with the left-hand side.

Use the identities for $\cos(x+y)$ *and* $\cos(x-y)$.

I was stuck here for about 15 minutes. Use the Pythagorean identities manipulated – then simplify.

Note: Problems 7 and 8 are from section 5.5.

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7. $\sec 2\theta = \dfrac{\sec^2 \theta}{2 - \sec^2 \theta}$

Work with the right-hand side.

$= \dfrac{\dfrac{1}{\cos^2 \theta}}{2 - \dfrac{1}{\cos^2 \theta}}$

Write in terms of cosine

$= \dfrac{\left(\dfrac{1}{\cos^2 \theta}\right)}{\left(2 - \dfrac{1}{\cos^2 \theta}\right)} \cdot \dfrac{\cos^2 \theta}{\cos^2 \theta}$

Multiply by $\cos^2 \theta$ *to clear fractions.*

$= \dfrac{1}{2\cos^2 \theta - 1}$

$= \dfrac{1}{\cos 2\theta}$

Use the identity $\cos 2\theta = 2\cos^2 \theta - 1$

$= \sec 2\theta$

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8. $(\cos x + \sin x)^2 = 1 + \sin 2x$

Work with the left-hand side.

$= \cos^2 x + 2\cos x \sin x + \sin^2 x$

$= \sin^2 x + \cos^2 x + 2\sin x \cos x$

Use the Pythagorean identity.

$= 1 + 2\sin x \cos x$

$= 1 + \sin 2x$