Mth 114 – Trigonometry – **Practice Exam 4 – Solutions**

1.

a. Since the range for $y = \cos^{-1} x$ is $[0, \pi]$, $y = \dfrac{5\pi}{6}$.

b. Since the range for $y = \tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $y = -\dfrac{\pi}{4}$.

c. Since the range for $y = \sec^{-1} x$ is $[0, \pi]$,

$$y = \sec^{-1} 2 \quad \Rightarrow \quad \sec y = 2 \quad \Rightarrow \quad \frac{1}{\cos y} = 2 \quad \Rightarrow \quad \cos y = \frac{1}{2}$$

$$\Rightarrow \quad y = \frac{\pi}{3}$$

2. Since the range for $y = \sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\theta = 45°$.

3. If we let $\theta = \sin^{-1}\left(-\dfrac{1}{5}\right) \quad \Rightarrow \quad \sin\theta = -\dfrac{1}{5}$, then using the Pythagorean Theorem, we can see that $r = \sqrt{24} = 2\sqrt{6}$. Then $\sec\theta = \dfrac{hyp}{adj} = \dfrac{5}{2\sqrt{6}} = \dfrac{5\sqrt{6}}{12}$.

4.

$$3\tan x - 1 = 2 \quad \Rightarrow \quad 3\tan x = 3 \quad \Rightarrow \quad \tan x = 1$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

So the solution set is $\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$

5.

$$2\sin^2 x + \sin x = 1 \quad \Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0 \quad \Rightarrow \quad (2\sin x - 1)(\sin x + 1) = 0$$

$$\Rightarrow \quad 2\sin x - 1 = 0 \quad \textit{or} \quad \sin x + 1 = 0$$

$$\Rightarrow \quad \sin x = \frac{1}{2} \qquad \sin x = -1$$

$$\Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$

So the solution set is $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}\right\}$.

6. $2\sin 3x = 1 \quad \Rightarrow \quad \sin 3x = \frac{1}{2}$

Since $0 \le x < 2\pi \quad \Rightarrow \quad 0 \le 3x < 6\pi$

So $3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

The solution set is then $\left\{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}\right\}$

7. $2\sec x + 1 = \sec x + 3 \quad \Rightarrow \quad \sec x = 2 \quad \Rightarrow \quad \cos x = \frac{1}{2} \quad \Rightarrow \quad x = 60°, 300°$

So the solution set is $\{60°, 300°\}$.

8.

$\sin\theta\cos\theta = \cos\theta \quad \Rightarrow \quad \sin\theta\cos\theta - \cos\theta = 0 \quad \Rightarrow \quad \cos\theta(1 - \sin\theta) = 0$

$\Rightarrow \quad \cos\theta = 0 \qquad or \qquad \sin\theta = 1$

$\theta = 90°, 270° \qquad\qquad \theta = 90°$

So the solution set is $\{90°, 270°\}$

9.

$y = \sin x - 2$

$y + 2 = \sin x$

$x = \sin^{-1}(y + 2)$

10.

$4\pi + 4\tan^{-1} y = \pi$

$4\tan^{-1} y = -3\pi$

$\tan^{-1} y = -\frac{3\pi}{4}$

Unfortunately, $-\frac{3\pi}{4}$ is not in the range of $y = \tan^{-1} x$, so there are no solutions to this equation.

11.

$$\sin^{-1} x + \tan^{-1} \sqrt{3} = \frac{2\pi}{3}$$

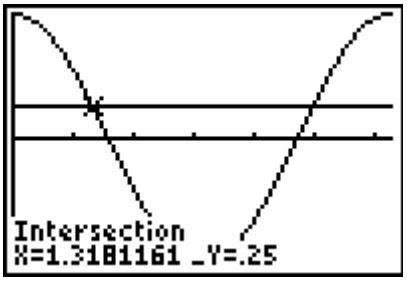
$$\sin^{-1} x + \frac{\pi}{3} = \frac{2\pi}{3}$$

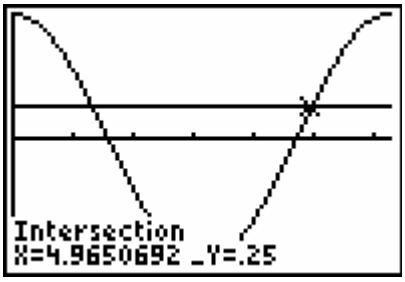
$$\sin^{-1} x = \frac{\pi}{3}$$

$$x = \sin\frac{\pi}{3}$$

$$x = \frac{\sqrt{3}}{2}$$

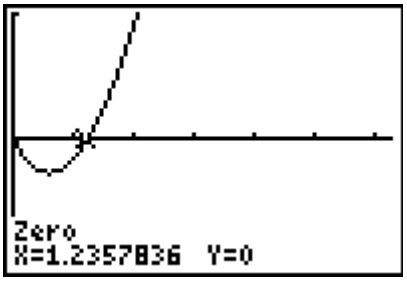
12.





As we can see on the screen captures, the solution set is $\{1.318116, 4.965069\}$.

13. Since $0^2 - 2 \cdot 0 + \sin 0 = 0$, $x = 0$ is one solution. To find the other, we need the graphing calculator.



From the screen shot, we can see that $x = 1.235784$ is the other solution. The solution set is then: $\{0, 1.235784\}$.