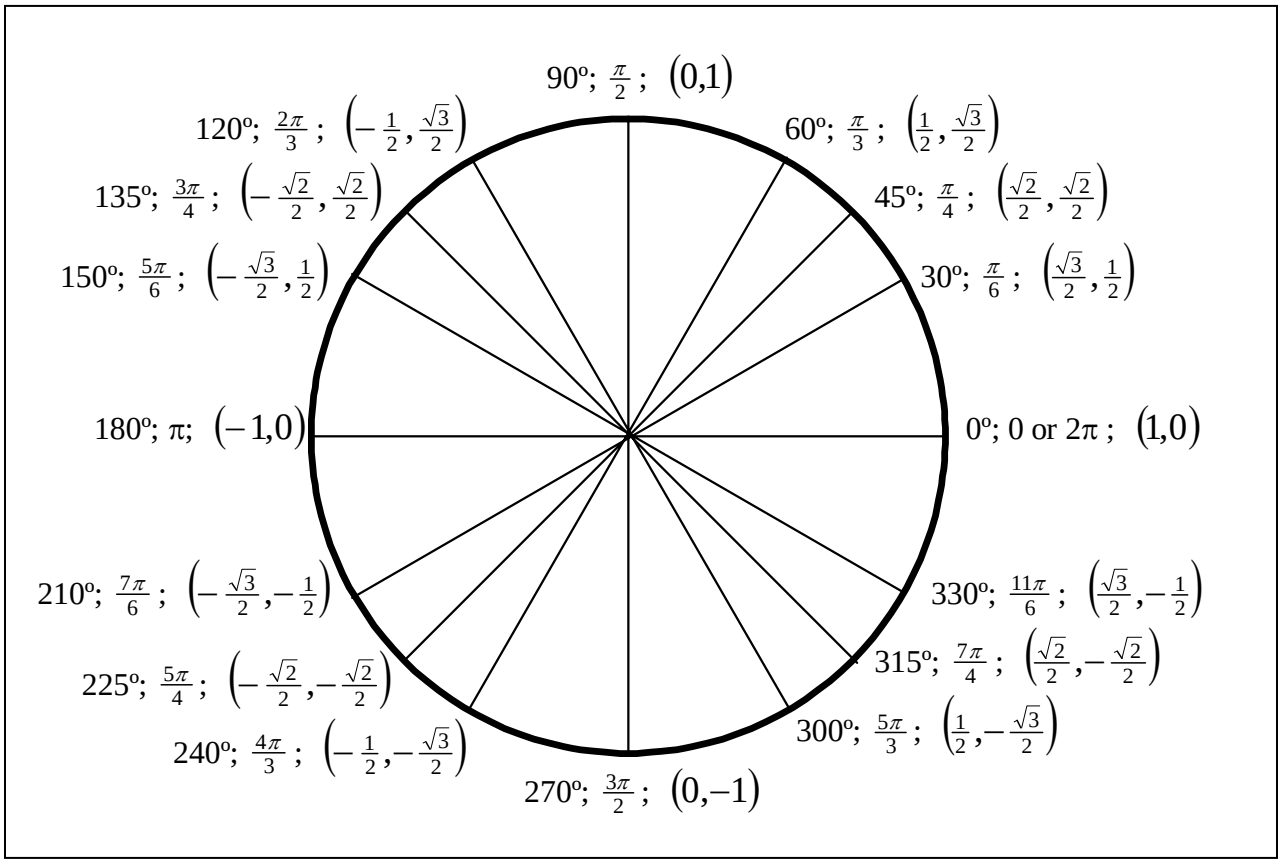
Mth 114 – Trigonometry – **Practice Exam 2 – Solutions**

NOTE: This exam should not be taken as a complete list of possible problems. It is merely intended to be an example of the length and difficulty level of the regular exam. To best utilize it as a *practice* exam, give yourself 55 minutes with no distractions. Try to emulate the classroom environment as much as possible. **Calculators are NOT ALLOWED on this portion.**

1. Complete the unit circle below. Include the radian measure, degree measure, and coordinates for each point.



2. Find the exact value of each expression.

a. $\csc\frac{\pi}{4}$ $\quad \csc\frac{\pi}{4} = \frac{1}{\sin\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

b. $\cos\left(-\frac{2\pi}{3}\right)$ $\quad \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

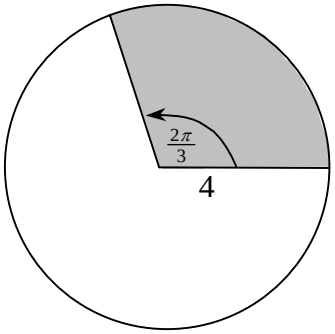
c. $\tan\frac{11\pi}{6}$

$$\tan\frac{11\pi}{6}=\frac{y}{x}=\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

d. $\sin\left(-\frac{7\pi}{6}\right)$

$$\sin\left(-\frac{7\pi}{6}\right)=\sin\frac{5\pi}{6}=\frac{1}{2}$$

3. Given the drawing below, answer the given questions.



a. Find the exact length of the arc intercepted by the given angle.

$$s=r\theta=4\cdot\frac{2\pi}{3}=\frac{8\pi}{3}$$

b. Find the exact area of the sector.

$$A=\frac{1}{2}r^2\theta=\frac{1}{2}\cdot4^2\left(\frac{2\pi}{3}\right)=\frac{16\pi}{3}$$

4. Find the exact value of s in $\left[\frac{\pi}{2},\pi\right]$ such that $\sin s=\frac{1}{2}$.

$\sin s=\frac{1}{2}=\frac{opp}{hyp}$ so the triangle has an opposite side of 1 and a hypotenuse of 2. This means the reference angle is $30°$ or $\frac{\pi}{6}$ so the angle (in radians) must be $\frac{5\pi}{6}$.

5. What is the angular speed of a person standing on the surface of the earth due to the rotation of the earth?

No matter where we are on the surface, we do a complete rotation in one day.

$$\omega=\frac{\theta}{t}=\frac{2\pi\text{ radians}}{1\text{ day}}=2\pi\text{ radians/day}$$

6. Find the period, amplitude, and phase shift for each trigonometric function.

a. $y = -2\sin\frac{1}{2}x$

Amplitude = 2, period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$, phase shift = 0. (no phase shift)

b. $y = 2 + \frac{1}{2}\cos(2x + \pi)$

Amplitude = $\dfrac{1}{2}$, period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{2} = \pi$, phase shift $= -\dfrac{\pi}{2}$

For the phase shift, notice that $\cos(2x + \pi) = \cos\left[2\left(x + \dfrac{\pi}{2}\right)\right]$

c. $y = -\cos\left(x + \frac{\pi}{2}\right)$

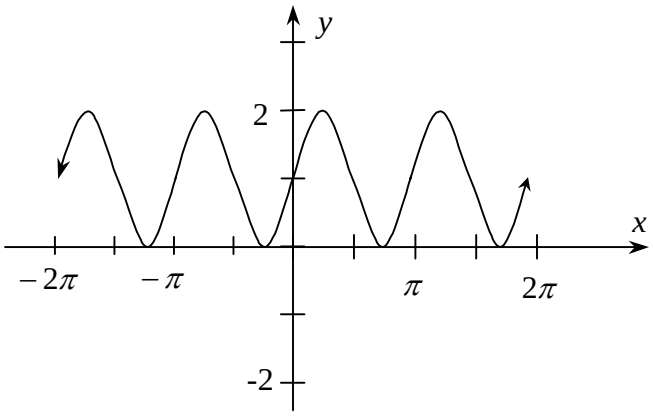
Amplitude = 1, period $= 2\pi$, phase shift $= -\dfrac{\pi}{2}$

d. $y = 3 + \sin\left(3x + \frac{\pi}{2}\right)$

Amplitude = 1, period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{3}$, phase shift $= -\dfrac{\pi}{6}$

For the phase shift, notice that $\sin\left(3x + \frac{\pi}{2}\right) = \sin\left[3\left(x + \dfrac{\pi}{6}\right)\right]$

7. Find a function of the form $y = c + a\cos[b(x - d)]$ whose graph is shown below.

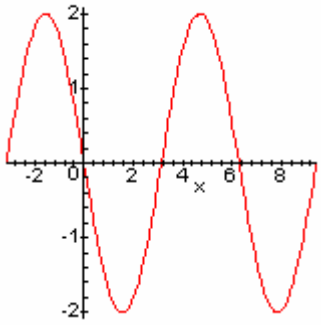


The graph is that of cosine, but shifted up 1, right $\dfrac{\pi}{4}$, with a period of π.

So $y = 1 + \cos\left[2\left(x - \dfrac{\pi}{4}\right)\right]$

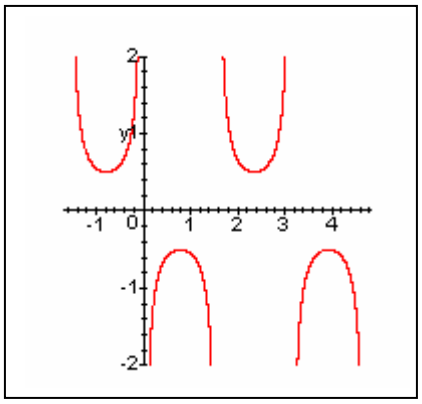
8. Graph each function over a two-period interval.

a. $y = 2\sin(x - \pi)$

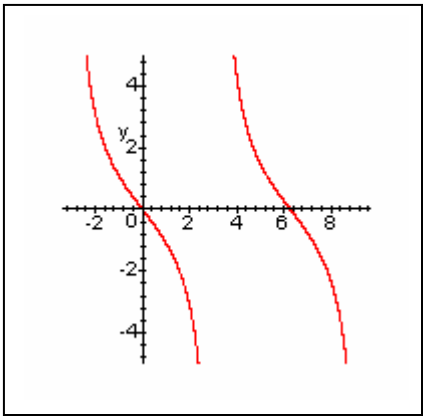


This is the output from some computer graphing software. I know the scale doesn't make sense, but the graph is just too hard to do in MS Word! (Just remember that $\pi \approx 3.14$.)

b. $y = \frac{1}{2}\sec\left[2x + \frac{\pi}{2}\right]$



c. $y = -2\tan\frac{1}{2}x$



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Calculators ARE ALLOWED on this portion.

9. The pulley shown has a radius of 12.96 cm. Suppose it takes 18 sec for 56 cm of belt to go around the pulley.
   a. Find the angular speed of the pulley in radians per second.

To determine ω, we need to first find θ. Since $s = r\theta, \theta = \frac{s}{r} = \frac{56}{12.96} \approx 4.32$ radians.

Then $\omega = \frac{\theta}{t} = \frac{4.32 \text{ radians}}{18 \text{ sec}} \approx 0.24$ radians/sec

   b. Find the linear speed of the belt in centimeters per second.

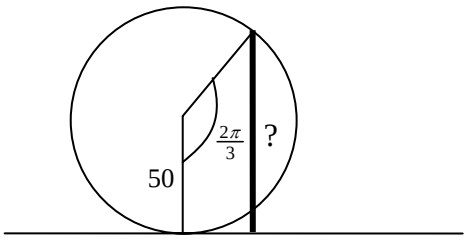
$v = r\omega = (12.94 \text{ cm})(0.24 \text{ radians/sec}) \approx 3.1 \text{ cm/sec}$

10. The hour hand of a wall clock measures 6.0 in. from its tip to the center of the clock. What distance does the tip of the hour hand travel during the time period from 1:00 to 3:00?

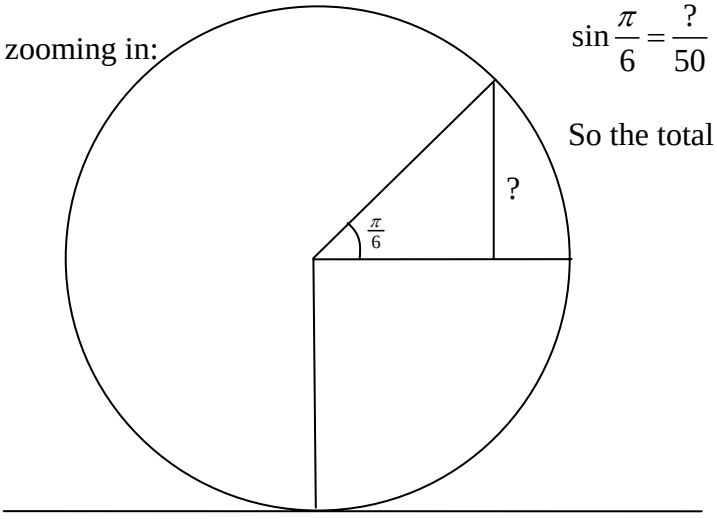
We know $s = r\theta$, and we know $r = 6$ in, so we just need to find θ. From 1:00 to 3:00 is 2 hrs, or $\frac{2}{12} = \frac{1}{6}$ of a rotation. $\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$ so $\theta = \frac{\pi}{3}$. Therefore,

$s = r\theta = (6 \text{ in})\left(\frac{\pi}{3}\right) = 2\pi \text{ in} \approx 6.3 \text{ in}$.

11. A Ferris wheel has radius 50 ft. A person takes a seat and then the wheel turns $\frac{2\pi}{3}$ radians. How far is the person above the ground?



zooming in:



The primary challenge on this problem is clarifying how to set up the drawing. As we can see here, we need to find the distance above the center of the Ferris wheel (see 2nd picture) and then add 50 ft.

$\sin\frac{\pi}{6} = \frac{?}{50} \quad \Rightarrow \quad ? = 50\sin\frac{\pi}{6} = 25$

So the total distance above the ground is 75 ft.