Mth 102 – General Education Statistics – Practice Exam 5 – Solutions

1. The confidence interval in letter *b* will have a higher confidence since it is wider. A wider confidence interval means we can be more confident that our interval contains the true population mean.

2. We should first determine if the data are normally distributed by doing a normal probability plot. We also need to check if there are outliers, which is easy to see on a modified box plot.

3. Since we know σ, we use the *Z*-interval: $\bar{x} \pm Z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$.

   95% CI $\Rightarrow$ $\alpha = 0.05$ $\Rightarrow$ $Z_{\alpha/2} = 1.96$ and then

   $$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad 33.4 \pm 1.96 \cdot \frac{42}{\sqrt{32}} \quad \Rightarrow \quad (18.8, 48.0)$$

   We are 95% confident that the mean duration of imprisonment for patients with chronic PTSD was between 18.8 and 48.0 months.

4.
   a. Yes, since we have σ and the population is normally distributed.

   b. No, since $n < 30$ and the data are not normally distributed.

   c. Yes – even though the data are not normally distributed, but our sample size is more than 30.

   d. Yes – not only do we have 30 in our sample, the data look to be normally distributed without outliers.

5. Since we know σ, we use the *Z*-interval: $\bar{x} \pm Z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$.

   95% CI $\Rightarrow$ $\alpha = 0.05$ $\Rightarrow$ $Z_{\alpha/2} = 1.96$ and then

   $$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad 63.4 \pm 1.96 \cdot \frac{2.4}{\sqrt{10}} \quad \Rightarrow \quad (61.9, 64.9)$$

   We are 95% confident that the mean height of women is between 18.8 and 48.0 in.

6. Using $n = \left(\dfrac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$, with $Z_{\alpha/2} = 1.96$, $\sigma = 7.7$, and $E = 3$, we see that $n \approx 25.3$, so we need at least 26 students in our sample.

7. Again, we know σ, we use the Z-interval: $\bar{x} \pm Z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$.

   95% CI $\Rightarrow \alpha = 0.05 \Rightarrow Z_{\alpha/2} = 1.96$ and then

   $$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad 22.46 \pm 1.96 \cdot \frac{4.10}{\sqrt{27}} \quad \Rightarrow \quad (20.91, 24.01)$$

8. In this case, we do not know σ, so we use the t-interval: $\bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$.

   99% CI $\Rightarrow \alpha = 0.01 \Rightarrow t_{\alpha/2} = t_{0.005,\,25} = 2.787$ and then

   $$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad \Rightarrow \quad 68.1 \pm 2.787 \cdot \frac{7.78}{\sqrt{26}} \quad \Rightarrow \quad (63.8, 72.4)$$

9. Since the standard deviation we are given is from the study, we can assume that we do not know σ, so we use the t-interval: $\bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$.

   90% CI $\Rightarrow \alpha = 0.1 \Rightarrow t_{\alpha/2} = t_{0.05,\,21} = 1.721$ and then

   $$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad \Rightarrow \quad 19.6 \pm 1.721 \cdot \frac{5.8}{\sqrt{22}} \quad \Rightarrow \quad (17.5, 21.7)$$

10. They are wider because we are using an estimate (s) for a parameter (σ), giving us less confidence about our results.

11. Again, we do not know σ, so we use the t-interval: $\bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$.

    95% CI $\Rightarrow \alpha = 0.05 \Rightarrow t_{\alpha/2} = t_{0.025,\,9} = 2.262$ and then

    $$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad \Rightarrow \quad 2.33 \pm 2.262 \cdot \frac{2.002}{\sqrt{10}} \quad \Rightarrow \quad (0.90, 3.76)$$

12. About 950 (95% of 1000) should contain the actual population mean.