Fraction Review

A *fraction* is a ratio of two integers. The top term is referred to as the *numerator*, and the bottom as the *denominator*, so the fraction $\frac{3}{4}$ has a numerator of 3 and a denominator of 4.

Multiplication

To multiply fractions, multiply the numerators and multiply the denominators. After multiplying, be sure to simplify if possible. When multiplying by an integer, simply place it over 1 and multiply as before.

$$\text{rule: } \frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

ex: $\frac{3}{2}\cdot\frac{3}{5}=\frac{9}{10}$ ex: $\frac{2}{5}\cdot\frac{9}{8}=\frac{18}{40}=\frac{9\cdot 2}{20\cdot 2}=\frac{9}{20}$ ex: $3\cdot\frac{4}{5}=\frac{3}{1}\cdot\frac{4}{5}=\frac{12}{5}$

Division

To divide fractions, just multiply by the *reciprocal*. (The *reciprocal* of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The reciprocal of an integer a is $\frac{1}{a}$.)

$$\text{rule: } \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$$

ex: $\frac{3}{2}\div\frac{3}{5}=\frac{3}{2}\cdot\frac{5}{3}=\frac{15}{6}=\frac{3\cdot 5}{3\cdot 2}=\frac{5}{2}$ ex: $\frac{3}{4}\div 2=\frac{3}{4}\cdot\frac{1}{2}=\frac{3}{8}$ ex: $\frac{\frac{6}{7}}{\frac{2}{3}}=\frac{6}{7}\cdot\frac{3}{2}=\frac{18}{14}=\frac{9\cdot 2}{7\cdot 2}=\frac{9}{7}$

Addition and Subtraction

In order to add or subtract fractions, the fractions must first have *common denominators*. (Fractions have *common denominators* if they have the same denominator.) If two fractions have common denominators, they can be added by adding their numerators while keeping the denominator the same.

$$\text{rule: } \frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \qquad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

ex: $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}$ ex: $\frac{7}{6}-\frac{17}{6}=\frac{-10}{6}=-\frac{5\cdot 2}{3\cdot 2}=-\frac{5}{3}$

ex: $\frac{3}{4}+\frac{5}{6}=\left[\frac{3}{4}\cdot\frac{3}{3}\right]+\left[\frac{5}{6}\cdot\frac{2}{2}\right]=\left[\frac{9}{12}\right]+\left[\frac{10}{12}\right]=\frac{19}{12}$ ex: $\frac{3}{5}-\frac{4}{15}=\left[\frac{3}{5}\cdot\frac{3}{3}\right]-\frac{4}{15}=\left[\frac{9}{15}\right]-\frac{4}{15}=\frac{5}{15}=\frac{5}{5\cdot 3}=\frac{1}{3}$

Practice Problems:

1. $\frac{3}{4} \cdot \frac{2}{5}$

2. $\frac{1}{3} \cdot \frac{8}{5}$

3. $\frac{9}{5} \cdot \frac{10}{21}$

4. $\frac{3}{4} \div \frac{2}{5}$

5. $6 \div \frac{1}{3}$

6. $\frac{2}{5} \div \frac{5}{8}$

7. $\frac{\frac{1}{2}}{8}$

8. $\frac{1}{3} \div \frac{8}{9}$

9. $\frac{3}{4} + \frac{1}{2}$

10. $4 + \frac{3}{5}$

11. $\frac{1}{5} - \frac{8}{3}$

12. $\frac{21}{4} - 3 + \frac{5}{6}$

Solutions: 1. $\frac{3}{10}$ 2. $\frac{8}{15}$ 3. $\frac{6}{7}$ 4. $\frac{15}{8}$ 5. 18 6. $\frac{16}{25}$ 7. $\frac{1}{16}$ 8. $\frac{3}{8}$ 9. $\frac{5}{4}$ 10. $\frac{23}{5}$ 11. $-\frac{37}{15}$ 12. $\frac{37}{12}$