Mth 096 – Beginning Algebra – **Practice Exam 6 – Solutions**

Special Factoring Formulas:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

1.

a. $12y^4 = 4 \cdot 3 \cdot y^4 = 2 \cdot 2 \cdot 3 \cdot y^4$

$20y^3 = 4 \cdot 5 \cdot y^3 = 2 \cdot 2 \cdot 5 \cdot y^3$

From these, we can see that the GCF = $2 \cdot 2 \cdot y^3 = 4y^3$

b. $18x^2y = 9 \cdot 2 \cdot x^2y = 3 \cdot 3 \cdot 2 \cdot x^2y$

$9x^3y^3 = 3 \cdot 3 \cdot x^3y^3$

$36x^3y^2 = 9 \cdot 4 \cdot x^3y^2 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot x^3y^2$

Looking at what is in common, we see the GCF = $3 \cdot 3 \cdot x^2y = 9x^2y$

2. $24a^3b - 36a^2c^2 + 48ab^3 = 12a(2a^2b - 3ac^2 + 4b^3)$

3. There are 4 terms, so use grouping:
$x^2 - 8x - xy + 8y = x(x-8) - y(x-8) = (x-8)(x-y)$

4. $10xy - 15y^2 = 5y(2x - 3y)$

5. This is a *sum* of squares, with no common factors, so it is *prime*.

6. This is a *difference* of squares: $a^2 - 36 = a^2 - 6^2 = (a+6)(a-6)$

7. There are 3 terms, with no common factors, with a leading coefficient that is not 1. Using the $a \cdot c$ strategy: $a \cdot c = 3 \cdot 6 = 18$, and then $2 \cdot 9 = 18, 2 + 9 = 11$, so:
$3y^2 + 11y + 6 = 3y^2 + 2y + 9y + 6 = y(3y+2) + 3(3y+2) = (3y+2)(y+3)$

8. Even though there are both *x* and *y* variables here, this is just the strategy where we look for factors of –10 whose sum is 3: $x^2 + 3xy - 10y^2 = (x+5y)(x-2y)$

9. There are 3 terms with a leading coefficient of 1, so look for factors of 30 whose sum is –13:
$x^2 - 13x + 30 = (x-3)(x-10)$

10. First, there is a common factor of 2: $2t^2 + 20t + 50 = 2(t^2 + 10t + 25)$
Then the remaining factor is of the form $a^2 + 2ab + b^2$, so $2(t^2 + 10t + 25) = 2(t+5)^2$

11. This looks fairly messy, but we can see that both the first and last terms are something squared. We just need to determine whether the middle term is 2 times their product:

$$25x^2 + 60xy + 36y^2 \overset{?}{=} (5x)^2 + 2(5x)(6y) + (6y)^2$$

Since $2(5x)(6y) = 60xy$, we *can* factor this trinomial: $25x^2 + 60xy + 36y^2 = (5x + 6y)^2$

12. There are no common factors, and this is a difference of squares:

$$25a^2 - 9b^2 = (5a)^2 - (3b)^2 = (5a + 3b)(5a - 3b)$$

13. This is a sum of *cubes*, so using the special factoring formula given on page 1:

$$y^3 + 125 = y^3 + 5^3 = (y + 5)(y^2 - y \cdot 5 + 5^2) = (y + 5)(y^2 - 5y + 25)$$

14. We must first set the equation equal to 0, then factor, and finally set each factor equal to 0.

$$a^2 = 5a \quad \Rightarrow \quad a^2 - 5a = 0 \qquad a(a - 5) = 0$$

Setting each factor equal to 0, we have: $a = 0$ or $a - 5 = 0$, so $a = 0, 5$

Don't forget to check in the original equation.

15. Since this is already equal to 0, we just factor and set each factor equal to 0.

$$x^2 - 3x - 10 = 0 \qquad \Rightarrow \qquad (x - 5)(x + 2) = 0$$

Setting each factor equal to 0, we have: $x - 5 = 0$ or $x + 2 = 0$, so $x = -2, 5$

Again, don't forget to check in the original equation.

16. Like #14, we need to first get this equal to 0.

$$y^2 + y = 12 \qquad \Rightarrow \qquad y^2 + y - 12 = 0 \qquad \Rightarrow \qquad (y + 4)(y - 3) = 0$$

Setting each factor equal to 0, we have: $y + 4 = 0$ or $y - 3 = 0$, so $y = -4, 3$

Don't forget to check in the original equation.

17. To find the *x*-intercepts, we set $y = 0$.

$$y = 0 \quad \Rightarrow \quad x^2 + 4x - 5 = 0 \quad \Rightarrow \quad (x + 5)(x - 1) = 0$$

Solving for *x* by setting each factor equal to zero, we have *x*-intercepts of $x = -5, 1$.

18. When the ball hits the ground, its height above the ground is zero, so we set $h(t) = 0$ and solve for *t*.

$$h(t) = -5t^2 + 10t + 15 = 0 \quad \Rightarrow \quad -5(t^2 - 2t - 3) = 0 \quad \Rightarrow \quad -5(t - 3)(t + 1) = 0$$

$$\Rightarrow \quad t - 3 = 0 \quad \textit{or} \quad t + 1 = 0$$

$$\Rightarrow \quad t = 3 \quad \textit{or} \quad t = -1$$

Obviously, $t = -1$ is not a possibility, so $t = 3$ is the only answer. Therefore, we can say that the ball hit the ground 3 seconds after being thrown into the air.