MTH 098 – Intermediate Algebra

Courses with MTH 098 as a pre-requisite:
BEC 101 – Basic Economics
BEC 102 – Principles of Macroeconomics
BEC 103 – Principles of Microeconomics
BIO 105 – Survey of Environmental Biology
BIO 115 – Environmental Biology
CHM 101 – Preparatory Chemistry
CHM 112 – Elements of Chemistry: General
CIS 105 – Programming in BASIC
CIS 121 – Computer Science for Engineers
EGR 101 – General Engineering Drawing
GEO 115 – Introduction to Physical Geography
PHY 105 – Technical Physics
SPC 111 – Survey of Physical Science

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M. Rezac
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S. Trail
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Ratio and Proportion (Conversion Factors)

Without a doubt, the most common skill students need in Biology, Chemistry, Geology, and Physical Science is the ability to convert from one unit of measurement to another.

The basic principle used is that of ratios and proportions, but emphasis in those courses is on 'conversion factors'. Here are some examples you might consider using in your class.

ACC 105 (S. Hellmuth)

Problem: Land, buildings, and equipment are acquired for a lump sum of \$950,000. The market values of the three assets are, respectively, \$200000, \$500,000, and \$300000. What is the cost assigned to the equipment?

Solution: The cost of the equipment is its portion of the extra value. The market values sum to \$1,000,000, so the total extra cost was \$50,000. The equipment's portion of the total cost is $\frac{\$300{,}000}{\$1{,}000{,}000} = \frac{1}{3}$, so its portion of the extra value is $\frac{1}{3}$ of \$50,000, or \$16,667.

CHM 112 (M. Rezac)

Problem: How many moles are 1.204×10^{25} atoms of Gold?

Solution: Given 1 mole = 6.02×10^{23} atoms, $1.204 \times 10^{25} \text{ atoms} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} = 20 \text{ mol}$.

Problem: How many moles are in a sample of 0.0922 grams of chlorine gas?

Solution: Given the molecular weight of chlorine, (35.453)

$$0.0922 \text{ g} \cdot \frac{1 \text{ mol}}{35.453 \text{ g}} = 0.00260 \text{ mol}.$$

Note: One way to define molecular weight is the weight of one mole of the particle.

Problem: What is the mass of 5.0×10^{22} molecules of HI?

Solution: Given that 1 mole of HI = 128 g, and 1 mole = 6.02×10^{23} atoms,

$$5.0 \times 10^{22} \text{ molecules} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ molecules}} \cdot \frac{128 \text{ g}}{1 \text{ mol}} = 10.7 \text{ g}$$

<u>CHM 112/142</u> (S. Trail)

Problem: Alcohol (C_2H_6O) has a heat content of 29.3 kJ per gram. There are 4.184 J per calorie. How many Calories per gram does Alcohol have?

(*Students are supposed to know there are 1000 J in 1 kJ and 1000 cal in 1 Cal*.)

Solution: $\frac{29.3 \text{ kJ}}{1 \text{ g}} \cdot \frac{1000 \text{ J}}{1 \text{ kJ}} \cdot \frac{1 \text{ cal}}{4.184 \text{ J}} \cdot \frac{1 \text{ Cal}}{1000 \text{ cal}} = 7.003 \text{ Cal/g}$

Problem: Beer is 5 percent alcohol by volume. Express this as a ratio with units that include mL on the top and bottom.

Solution: $\frac{5 \text{ mL alcohol}}{1 \text{ mL beer}}$

<u>CIS 121</u> (G. Mayer)

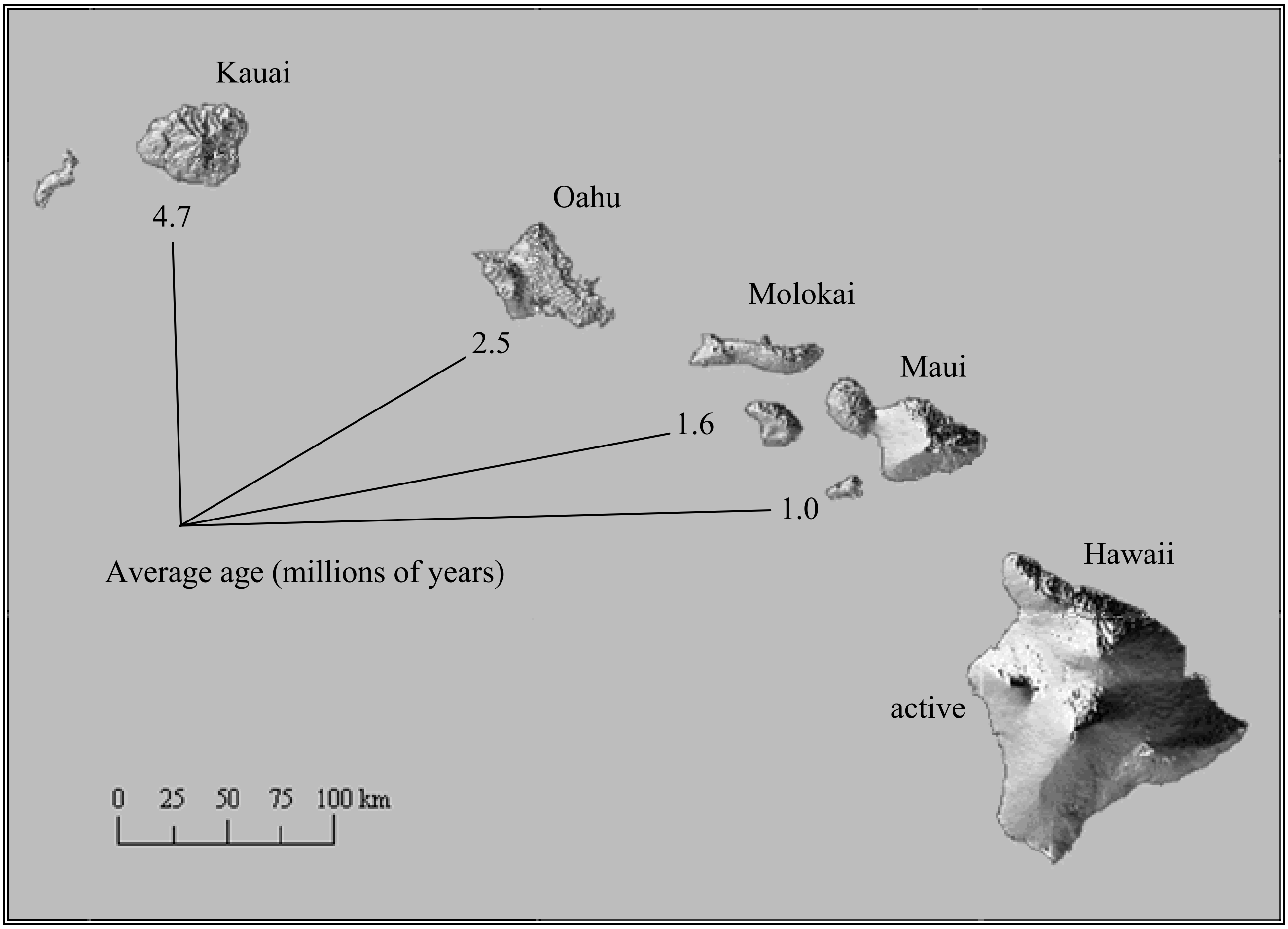
Problem: Write a program that will read in a weight in kilograms and will output the equivalent weight in pounds and ounces. There are 2.2046 pounds in a kilogram, and 16 ounces in a pound.

Solution: (or the mathematics portion of it)

First, compute $x \text{ kg} \cdot \frac{2.2046 \text{ lbs}}{1 \text{ kg}}$, then take the decimal portion of the result to compute the ounces: $y \text{ lbs} \cdot \frac{16 \text{ oz}}{1 \text{ lb}}$.

GEO 115 (T. Millen)

Problem: The figure below shows the age/distance relationship of the Hawaiian islands. Using any 2 points on this island chain, calculate the rate of movement of the Pacific Ocean plate in centimeters per year. The calculated rate of movement for the San Andreas Fault is 3 cm/yr. How does the rate for Hawaii compare to the San Andreas Fault rate?



SPC 111 (H. Wickersham)

Problem: How many meters is a 100 yd football field?

Solution: Given 0.305 m = 1 ft, $100 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{0.305 \text{ m}}{1 \text{ ft}} = 91.5 \text{ m}$.

Problem: A person weighs 135 lbs. What is their weight in grams?

Solution: Given 454 g = 1 lb, $135 \text{ lbs} \cdot \frac{454 \text{ g}}{1 \text{ lb}} = 61290 \text{ g}$.

Problem: You need 1250 ml of distilled water for a lab experiment. You have 1.25 quarts. Is that enough?

Solution: Given 1 L = 1.06 qt, $1250 \text{ ml} \cdot \frac{1 \text{ L}}{1000 \text{ ml}} \cdot \frac{1.06 \text{ qt}}{1 \text{ L}} = 1.325 \text{ qt}$. No, you do not have enough.

Alternately, $1.25 \text{ qt} \cdot \frac{1 \text{ L}}{1.06 \text{ qt}} \cdot \frac{1000 \text{ ml}}{1 \text{ L}} = 1179 \text{ ml}$, so you do not have enough.

Order of Operations

CHM 112/142 (S. Trail)

Steve wrote about calculator issues – that students don't know whether or not an answer makes sense. For example:

Problem: Find the average of 50 and 60.

According to Steve, "...more than a few will determine that the average is 80, (50 + 60 /2 = 80) *and make no comments to how ludicrous the answer is."*

CIS 121 (G. Mayer)

Problem: The term "wind chill" goes back to the Antarctic explorer Paul Siple, who coined it a 1939 dissertation, "Adaptation of the Explorer to the Climate of Antarctica." During the 1940s, Siple and Charles Passel conducted experiments on the time needed to freeze water in a plastic cylinder that was exposed to the elements. They found that the time depended on how warm the water was, the outside temperature, and the wind speed.

The following formula used to calculate wind chill was based on those experiments.

$$WC = 0.0817(3.71\sqrt{Wind} + 5.81 - 0.25Wind)(Temp - 91.4) + 91.4$$

where *Wind* is the current wind speed in miles per hour and *Temp* is the current Fahrenheit temperature.

In the fall of 2001, the U.S. National Weather Service replaced the formula with a new one. The new formula is based on greater scientific knowledge and on experiments that tested how fast the faces of volunteers cooled in a wind tunnel with various combinations of wind and temperature. The new formula is:

$$WC = 35.74 + 0.6215 \cdot Temp - 35.75 \cdot Wind^{0.16} + 0.4275 \cdot Temp \cdot Wind^{0.16}$$

(*Note: Glenn then asks his students to write a program to compute the wind chill for a certain temperature and wind speed, but this could easily be done as a simple evaluation in 098.*)

Problem: We would like to have a program that calculates the gas mileage (number of miles per gallon) of our car. Write a program that asks the user to enter the number of miles that they traveled and the number of gallons that they used. The program will then calculate the gas mileage that they had for that trip.

(*Note: Again, even though this is asking for a program, students are expected to know how to calculate gas mileage. Many CIS 121 students do not.*)

Solving Equations for Unknowns

<u>CHM 112/142</u> (S. Trail)

Going back to ratios, Steve mentioned that his students are "hopeless" when it comes to solving for an unknown. For example:

Problem: Solve for *x*.

$$\frac{6{,}200 \text{ cal}}{1 \text{ peanut}} = \frac{26{,}000 \text{ cal}}{x \text{ peanuts}}$$

Problem: Solve the Ideal Gas Law ($PV = nRT$) for V.

<u>CHM 112</u> (M. Rezac)

Problem: A party balloon is filled with helium at $T = 25°$ C under normal pressure $p = 100{,}000$ Pa having a volume of 2 liters. The balloon then rises to 10,000 ft where the temperature is $-20°$ C, pressure $p = 35{,}000$ Pa. What would be the volume?

Solution: *Students are to know the combined gas law:* $\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2}$.

After first converting the temperature from Celsius to Kelvin, we have the following information:

<u>ground</u>	<u>high altitude</u>
$p = 100{,}000$ Pa	$p = 35{,}000$ Pa
$T = 298$ K	$T = 253$ K
$V = 2$ L	$V =$ unknown (x)

ground		*high altitude*
$\frac{p_1 \cdot V_1}{T_1}$	=	$\frac{p_2 \cdot V_2}{T_2}$
$\frac{100{,}000 \cdot 2}{298}$	=	$\frac{35{,}000 \cdot x}{253}$

If we simplify each side as much as possible, we get: $671.14 = 138.34x$. Solving for *x*, we see that the volume at 10,000 ft is approximately 4.85 L.

Basic Calculations

ACC 105 (S. Hellmuth)

Problem: Davis Company's sales for July 16 were $10,800. Davis is required to collect a 7.5% state sales tax. What was the total cash received from customers?

BEC 102/103 (L. Garcia)

Problem: Given that the public debt in 2005 was $8 trillion and the GDP was $12.4 trillion, find the public debt as the percentage of GDP.

Problem: Given the price index of 180 for a product in December 2004, and a price index of 196 for the same product in December 2005, find the absolute change and the relative change of the price index.

BEC 102 (G. Stevens)

Problem: Use the information in the table to answer the questions.

	Government Spending	Tax Revenues	GDP
Year 1	$800	$825	$4,000
Year 2	850	850	4,200
Year 3	900	875	4,350
Year 4	950	900	4,500
Year 5	1,000	925	4,600

All data are in billions of dollars

In which year was the **budget deficit** $75 billion?

Assume that Year 1 is the first year for this economy and Year 5 is the current year. What is the **public debt** of this economy?

CHM 112 (M. Rezac)

Problem: What percent of the compound $C_6H_{12}O_6$ is carbon?

Solution: The atomic number of carbon is 6, so it has 6 protons and 6 neutrons. Hydrogen only has a proton in its nucleus, and oxygen has an atomic number of 8, with 8 protons and 8 neutrons. The total mass of the compound is thus $6 \cdot 12 + 12 \cdot 1 + 6 \cdot 16 = 180$, with carbon contributing 72, thus the percentage of the compound that is carbon is $\frac{72}{180} \cdot 100 = 40\%$.

GEO 115 (T. Millen)

Problem: Convert $22°$ F to Celsius.

Notation

BEC 102/103 (L. Garcia)

$$Price\ index = \frac{p_{al} \cdot w_a + p_{bl} \cdot w_b + p_{cl} \cdot w_c}{p_{abase} \cdot w_a + p_{bbase} \cdot w_b + p_{cbase} \cdot w_c} \cdot 100$$

Where
p_i is the price of good i
w_i is the relative importance of good i in the typical consumer's expenditures

Functions

<u>BEC 102/103</u> (L. Garcia)

Leticia wrote that her students need to understand function notation, where functions can depend on more than one variable. She also wrote that her students need to understand whether there is a positive or negative relationship between the dependent and independent variables. (Of course that also implies that students need to know what the dependent *and* independent *variables are.)*

Some function examples from BEC 102/103:

Demand function:
$Q_{dx} = f$ (Price$_x$, Price$_y$, Consumers' income, consumers' expectations,…)

Consumption function
$C = f$ (Income,…)

Production function
$Q = f$ (amount of labor, amount of capital,…)

Graphs/Plotting

<u>BEC 102/103</u> (L. Garcia)

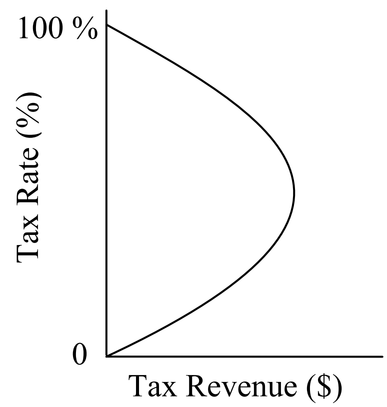
Rather than specific examples, Leticia gave some terms and ideas that students should be familiar with concerning graphs. Some of those were:

- Scatter diagram
- Positive linear relationship
- Negative linear relationship
- Positive non-linear relationship
- Negative non-linear relationship
- Slope
- Maximum and minimum

<u>BEC 102</u> (G. Stevens)

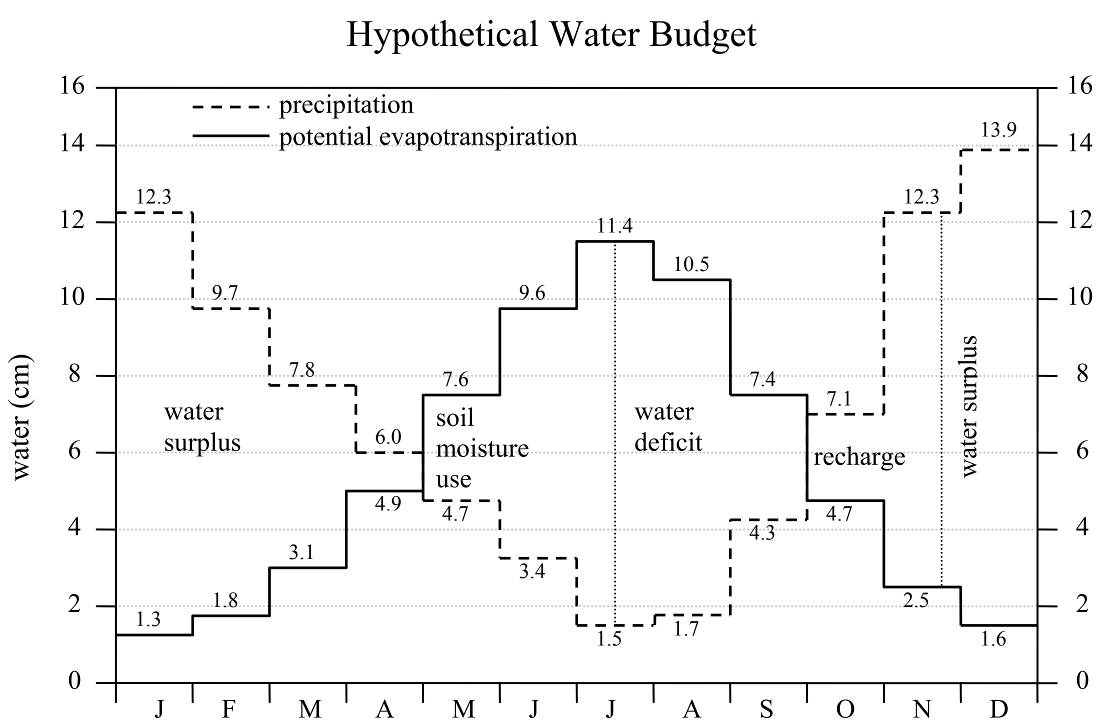
Problem: The diagram to the right describes the notion that as tax:

a. revenues increase from zero to 100%, tax rates will increase from zero to some maximum level then decline to zero.
b. rates increase from zero to 100%, tax revenue will increase from zero to some maximum level and decline to zero.
c. rates decrease from 100% to zero, tax revenue will decrease from 100% to some maximum level.
d. rates increase from zero to 100%, tax revenue will increase from zero to a maximum level.



Problem: The concept of "effective precipitation" refers to the availability of precipitation for plant growth. If precipitation that falls evaporates quickly because of intense radiation and high temperatures, it is not "effective" since it has never been available to plants. One way of elucidating the concept of effective precipitation is by determining a parameter called "potential evapotranspiration (PET)". PET refers to the maximum possible amount of water that can be evaporated by the available energy if the water supply is not limited. Thus the magnitude of PET depends primarily upon net radiation. Net radiation can be roughly related to temperature, allowing temperature to be used to estimate PET. With the knowledge of PET and precipitation (P) for a location, a water budget can be established.

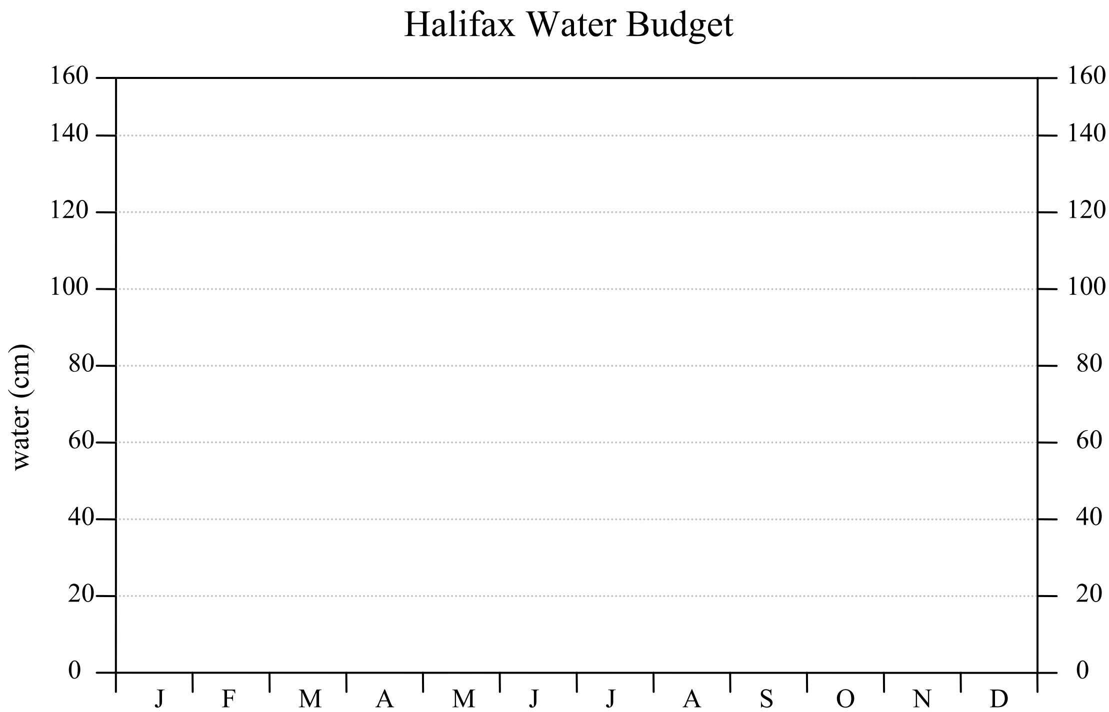
In developing a water budget or water balance diagram, storage of water in the soil must be considered. For months with PET less than P, a water surplus exists. If PET becomes greater than P, then soil moisture utilization must occur. If PET continues to be greater than P, a water deficit is established. If PET subsequently becomes less than P again, then a soil moisture recharge occurs until a water surplus is again established. Such a sequence of events is illustrated in the hypothetical example shown below:



Listed below are PET and P data for Halifax and Saskatoon, Canada. Halifax is located on the eastern coast of Canada, while Saskatoon is on the western interior plans approximately 500km east of the Rocky Mountains. On the provided charts, draw water budget diagrams for both of these locations using the same format shown in the chart above. Label the times of water surplus, soil moisture utilization, water deficit, and soil moisture recharge.

Halifax, Canada

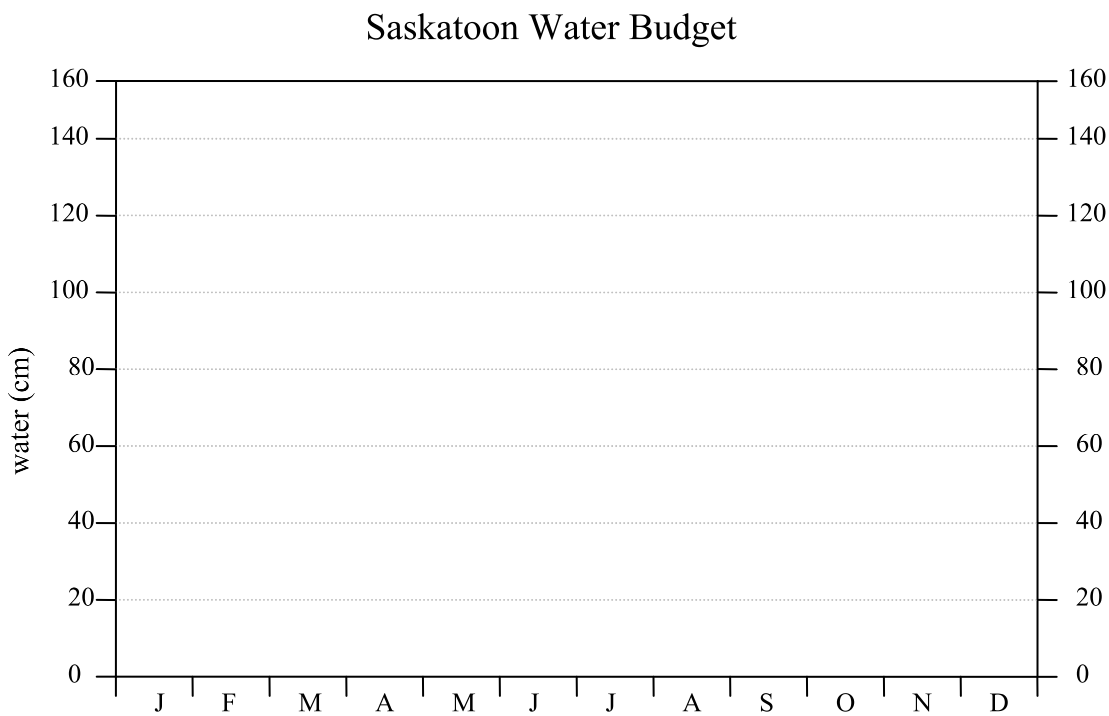
	J	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**	**O**	**N**	**D**	**Year**
PET (mm)	0	0	0	26	58	88	120	111	79	50	24	0	556
P (mm)	147	129	112	105	109	85	92	94	94	113	152	148	1380



How long does Halifax have a water deficit during the year?

Saskatoon, Canada

	J	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**	**O**	**N**	**D**	**Year**
PET (mm)	0	0	0	26	82	116	138	115	65	28	0	0	570
P (mm)	18	18	17	21	34	57	53	45	33	19	19	18	352



How long does Saskatoon have a water deficit during the year?

Lines

<u>ACC 105</u> (S. Hellmuth)

Problem: Professor John Morton has just been appointed chairperson of the Finance Department at Westland University. In reviewing the department's cost records, Professor Morton has found the following total cost associated with Finance 101 over the last several semesters:

Term	Number of Sections Offered	Total Cost
Fall, 2003	4	$10,000
Spring, 2004	6	$14,000
Summer, 2004	2	$ 7,000
Fall, 2004	5	$13,000
Spring, 2005	3	$ 9,500

Professor Morton knows that there are some variable costs such as amounts paid to graduate assistants, associated with the course. He would like to have the variable and fixed costs separated for planning purposes.

Use the high-low method, develop a cost formula ($Y = a + bX$)

Solution: *The high-low method finds the cost/item (slope) and uses that to find the fixed costs (y-intercept) using the highest and lowest activity levels.*

	Sections	Total Cost
high activity level	6	$14,000
low activity level	2	$ 7,000
change:	4	$ 7,000

The cost/section is then $\$7{,}000/4 = \$1750$, and the fixed cost is then $3500.
($\$14{,}000 - 6 * \$1{,}750 = \$3500$, or $\$7{,}000 - 2 * \$1{,}750 = \$3500$)

The cost formula is therefore: $Y = 3500 + 1750X$.

Problem: Jefferson Corporation acquired equipment on January 1, 2003 for $300,000. The equipment had an estimated useful life of 10 years and an estimated salvage value of $25,000. What is the book value on December 31, 2006 if Jefferson Corporation uses straight-line depreciation?

Interest

ACC 105 (S. Hellmuth)

Problem: Calculate the amount of interest due on a $3,000 12% 5 month note.

Solution: In this context, the interest is just simple interest, so we use $I = P \cdot r \cdot t$, where t is in years.

Thus, $I = 3000 \cdot 0.12 \cdot \frac{5}{12} = \$150$.