Magnetic Fields

Magnetism is closely related to electricity, but very different from a phenomenological perspective.

The absence of **magnetic monopoles** (single **N** or **S** 'charge') means there are asymmetries in the Maxwell Equation's.

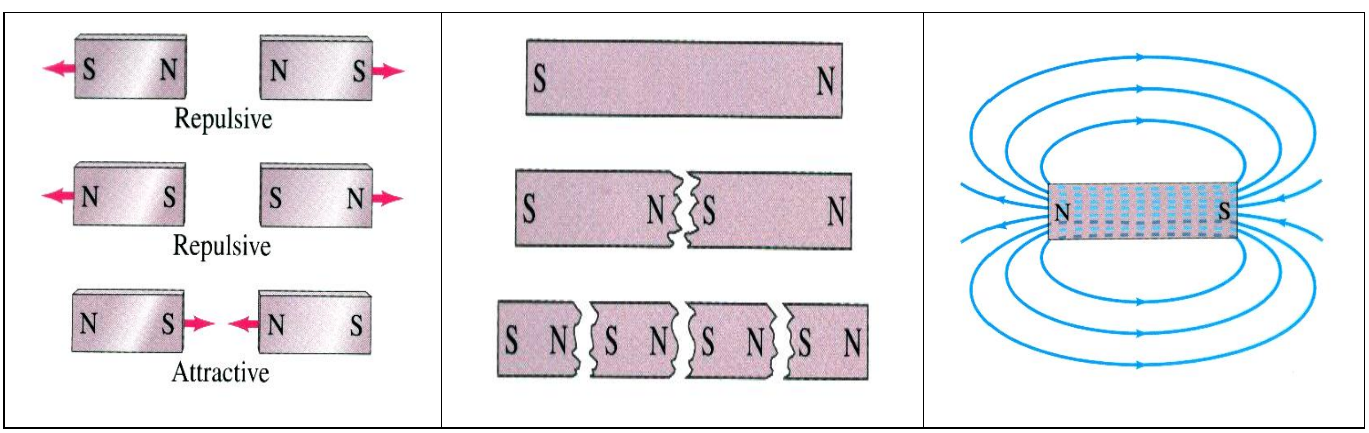
Since the **simplest magnetic structure is the magnetic dipole** (**N** and **S** exist only in pairs), **the net flux through any closed surface is always zero**.

In particular, electric field lines either diverge away from positive electric charge distributions or converge into negative electric charge distributions. Gauss' Law then relates the total net flux of these field lines to the charge enclosed as:

$$\Phi = \oint \vec{E} \bullet \hat{n} dA = q_{enclosed} / \varepsilon_0$$

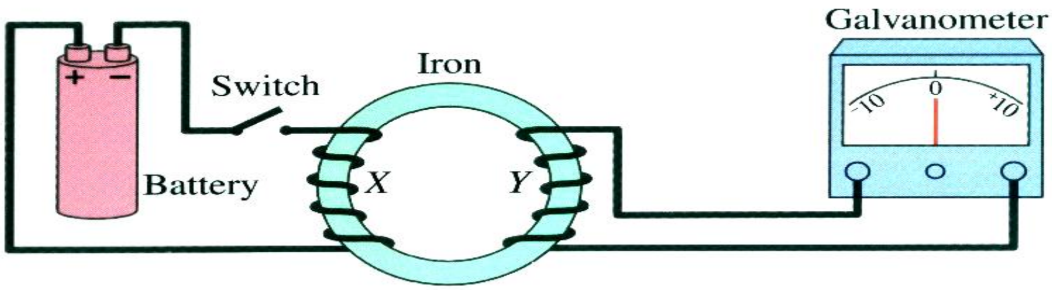
Magnetic field lines $\vec{B}$ always leave the 'north' but also always return to the 'south'.

$$\Phi = \oint \vec{B} \bullet \hat{n} dA = 0 \qquad B = d\Phi / dA_{\perp}$$



Magnetic fields occur naturally in minerals and ores when atomic **magnetic moments** align, and may be generated as with an **electromagnet** for example.

The utility of magnetic fields in industry and science principally stems from the fact that they may be generated using electricity and magnetic field time variations yield an Emf.



SI unit of magnetic field is the Tesla $1T = \frac{1N}{A \cdot m}$

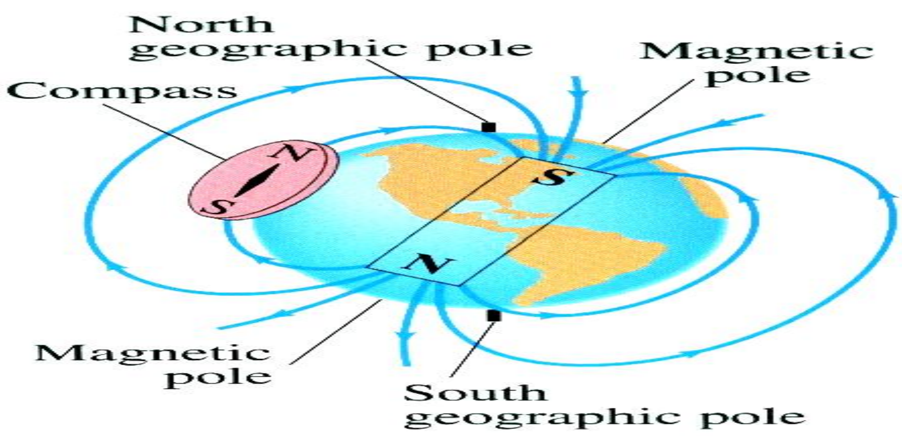
CGS unit of magnetic field is the Gauss $1 gauss = 10^{-4} T$

A **1A current carrying wire** produces a $2.0x10^{-7} T$ magnetic field at 1-m perpendicular distance from the wire.

Earth has an associated magnetic field resulting from a **dynamo effect** produced in the planet interior where molten material in motion ➔ currents ➔ magnetic fields.

The earth magnetic field is **~0.55 gauss** and magnets in use at Fermilab in Batavia are on the order of a few Tesla.

The direction of Earth's magnetic field is specified by the **field declination** and the **field inclination**. A compass can indicate the former and a dip meter the later.



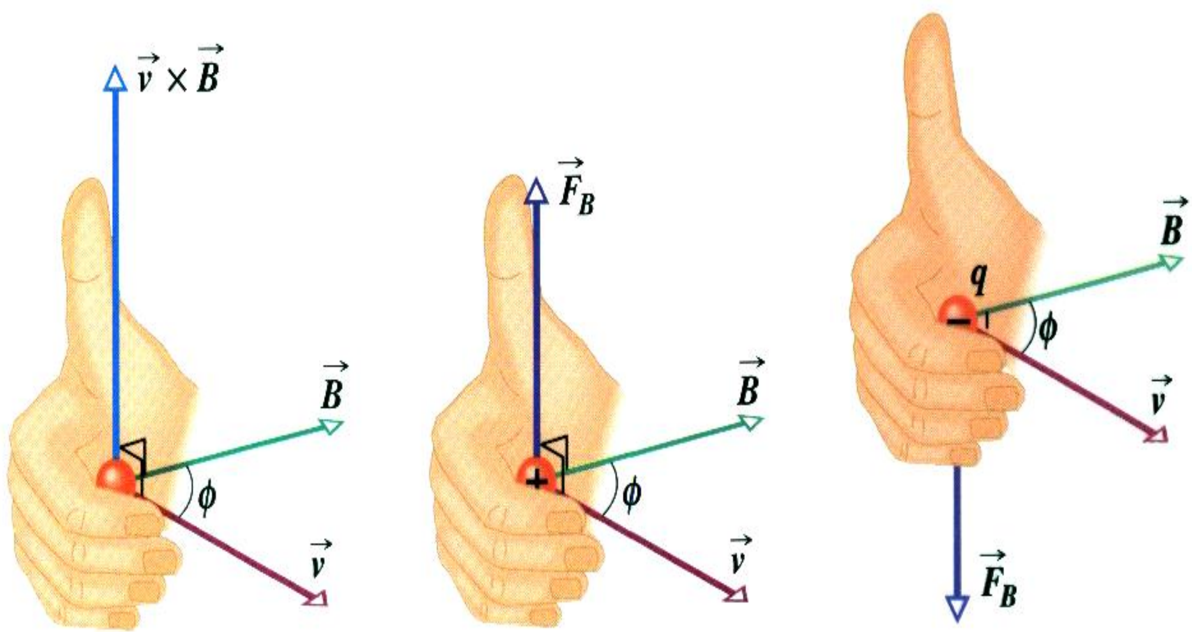
Magma deposits along the ocean floor near tectonic plate boundaries indicate a magnetic field reversal approximately every million years.

Magnetic Field

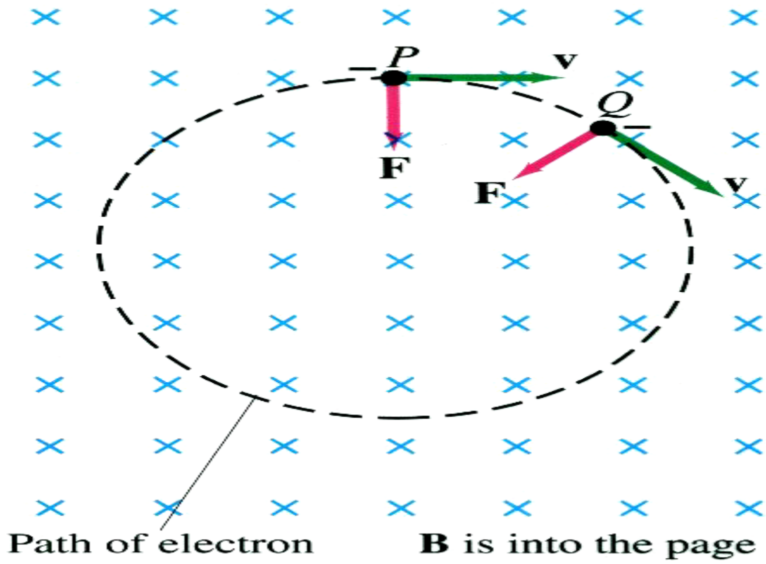
The force on a charged particle moving in a magnetic field is given by the cross product:

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{v} \ \ is \ \ the \ \ particles \ \ velocity \ \ vector$$

The magnetic force on a charged particle is therefore directed perpendicular to the plane formed by $\vec{v} \ \ and \ \ \vec{B}$



Notice that $\vec{F}_B$ **never** has a component parallel to $\vec{v}$ and therefore cannot change the particle's kinetic energy. $\vec{F}_B$ only changes the direction of $\vec{v}$.



In the above picture, the electron doesn't have any velocity vector component parallel to the magnetic field direction and therefore orbits in the **uniform circular motion** shown. Equating the centripetal force to the magnetic force to find the period of the electron:

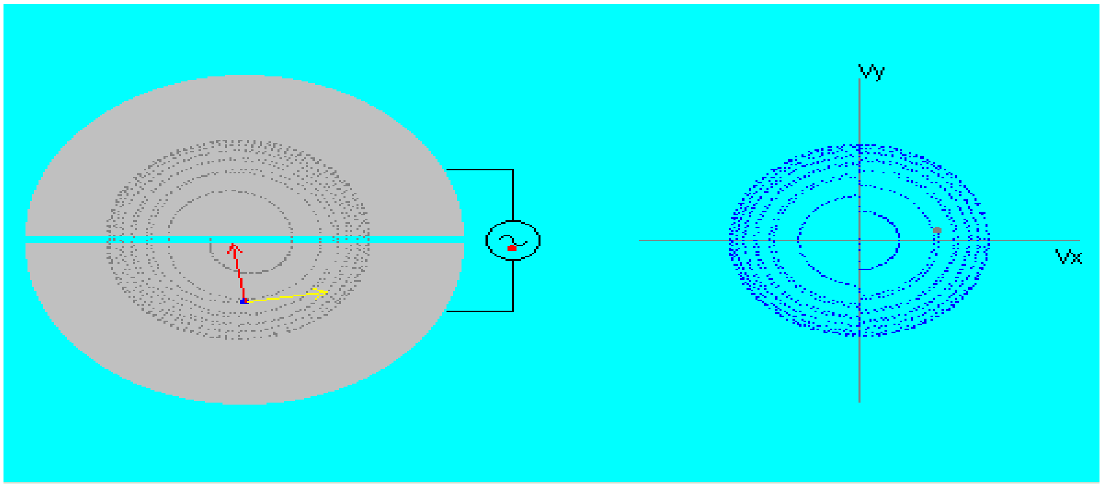
$$qvB = \frac{mv^2}{r} \qquad r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \qquad f = \frac{qB}{2\pi m}$$

All particles with the same $\frac{q}{m}$ have the same period.

The **cyclotron** **D**[s] shown below accelerate the injected charged particle each time the gap is crossed and its orbit is maintained by a magnetic field perpendicular to the page.

In order to accelerate the particle at each crossing, the potential on the **D**[s] must alternate at the orbit frequency: $2\pi m f_{Osc} = qB$

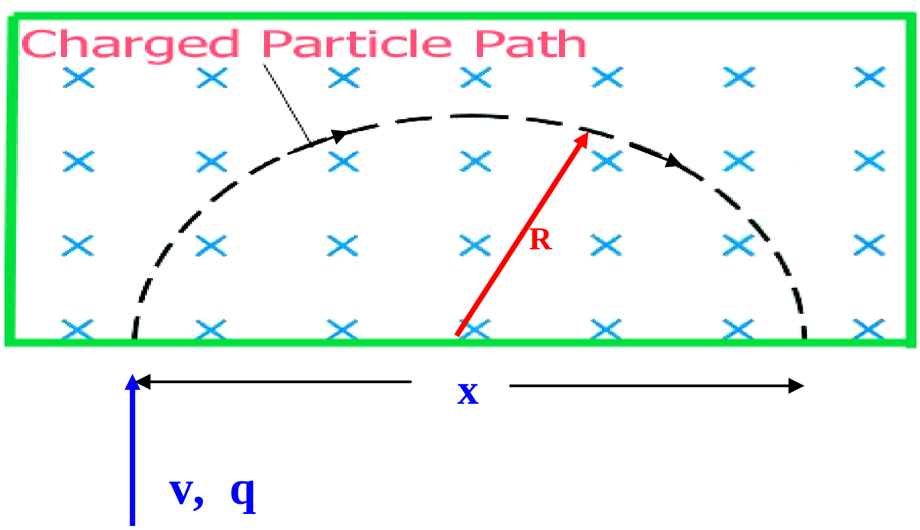


Notice that fixing $m,\ f_{Osc}\ \&\ q$ means one 'tunes' the cyclotron for maximal energy output by adjustments of the magnetic field.

Once the ratio of extracted high-energy charged particles to that of low energy injected particles is maximized, the tuning is complete.

Mass Spectrometer

The masses of charged particles may be determined by evaluating their radii of curvature once they're injected into a region of uniform magnetic field with a known velocity:



An electric potential imparts an initial velocity to the charged particle of:

$$qV = \frac{1}{2}mv^2$$

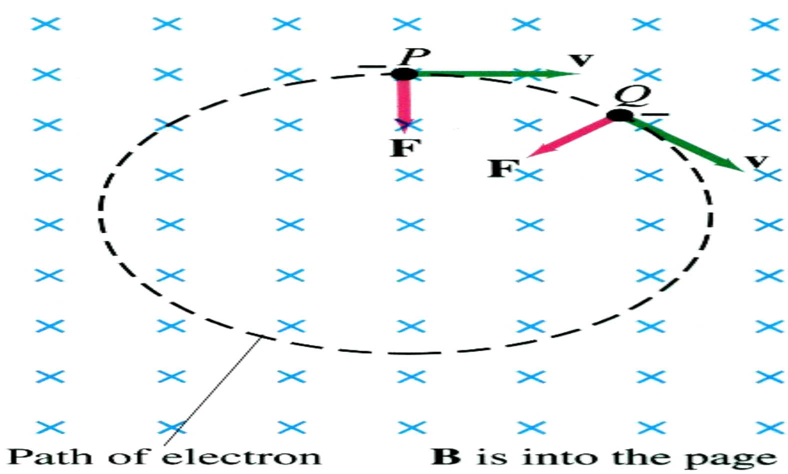
Solving for v ➔ $$v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{x}{2}$$

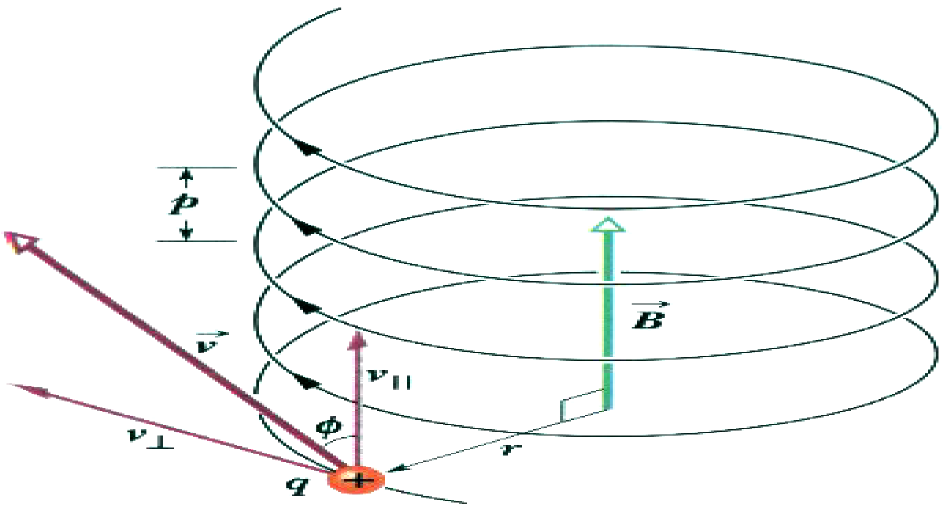
Solving for m ➔ $$m = \frac{B^2 q x^2}{8V}$$

Helical Trajectory

Circular paths resulted if the charged particle $\vec{v}$ was perpendicular to the applied $\vec{B}$



For a particle $\vec{v}$ with component parallel to the $\vec{B}$, then as it begins tracing a circular path it also moves out of the reentrant plane.



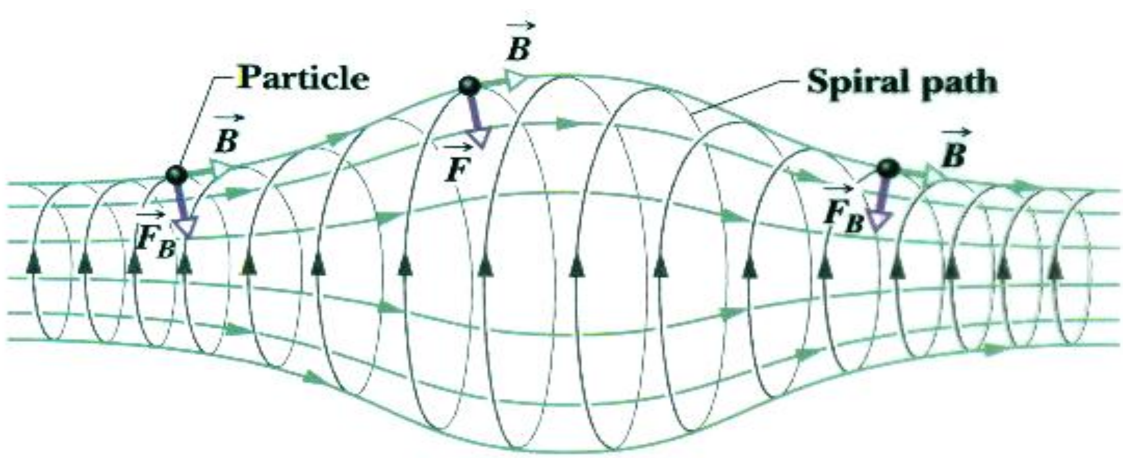
The **pitch** of the spiral is $v_{//}T$ where $T = \dfrac{2\pi m}{qB}$ is circular path period

$$p = vCos(\phi)\frac{2\pi m}{qB}$$

The angle ϕ lying between $\vec{v}$ and $\vec{B}$

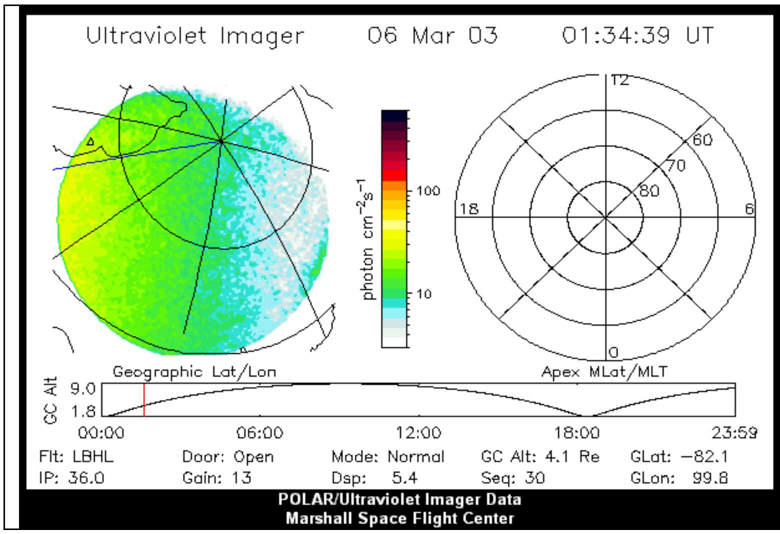
Magnetic Traps

A non-uniform magnetic field may be constructed to contain charged particles within a specific volume where they may be manipulated and researched. Such devices are known as **magnetic bottles** and are used in condensation experiments for example.



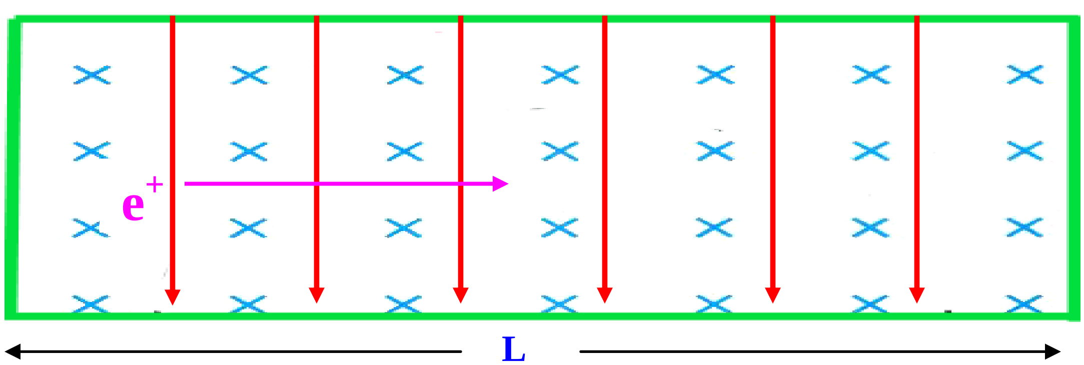
Existing above our atmosphere, the **Van Allen radiation belt** is formed out of solar wind particles trapped in earths magnetic field.

Overloaded by increased solar activity, the increased electric fields within the radiation belt drive charged particles from the trap down into the atmosphere where ionization and subsequent reformation of Oxygen and Nitrogen atoms lead to **Aurora Borealis**.



Crossed Fields

If both magnetic and electric fields are present in a region where a charged particle is in motion, then the total force on the particle is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$



$\vec{F}_B$ is up $\qquad \vec{F}_E$ is down

When these two forces balance, the positron motion is horizontal.

In the absence of a magnetic field the vertical position as a function of time will be:

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}\frac{L^2}{v^2}$$

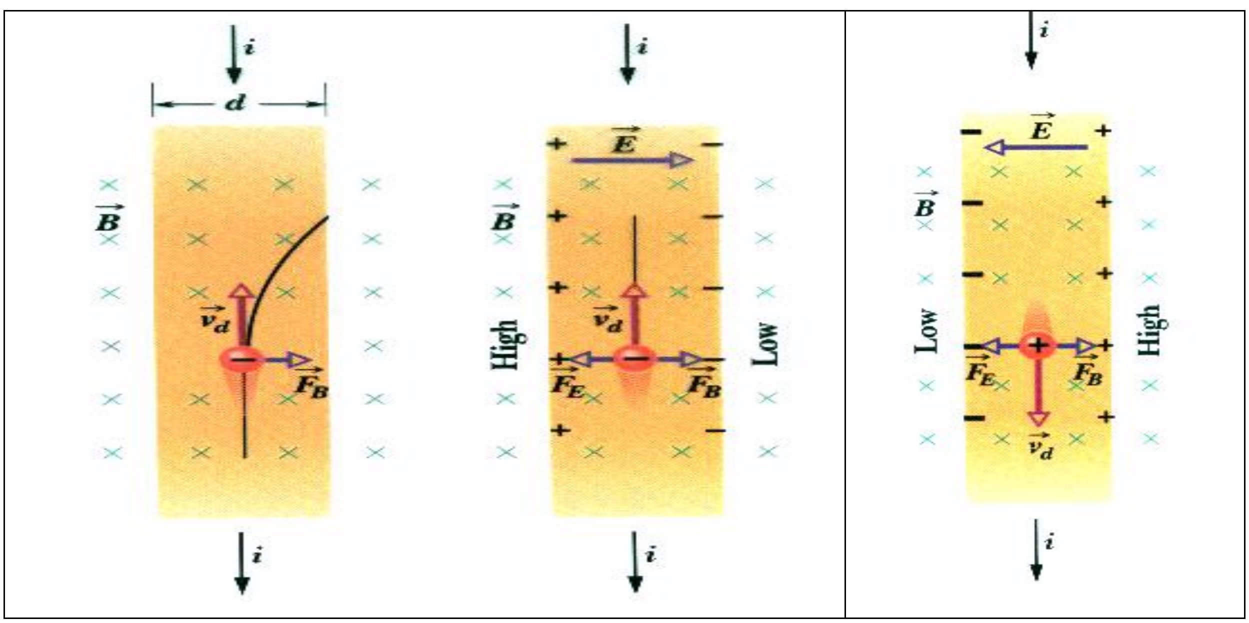
With both fields and at zero deflection $qE = qvB$ or $v = \frac{E}{B}$

Knowledge of the velocity allows the ratio m/q to be measured as was done in 1897 for the electron by J.J. Thomson.

$$\frac{m}{q} = \frac{EL^2}{2yv^2}$$

Hall Effect (1897)

Placing a current carrying conductor in a magnetic field will result in a deflection of the charge carriers that creates a potential difference across the width **(d)** of the conductor.



As charge carriers move towards one side of the conductor the $\vec{E}$ being established increases in magnitude and the electric force eventually balances out the magnetic force.

$$eE = ev_d B$$

Since the charge carrier density is

$$n = \frac{i}{ev_d A}$$

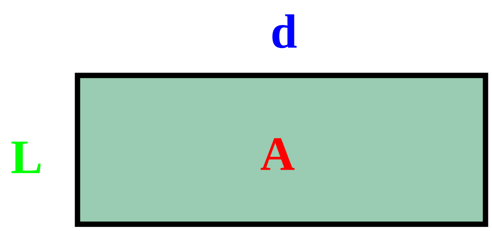
And

$$v_d = \frac{E}{B} = \frac{V}{Bd}$$

Charge carrier density is

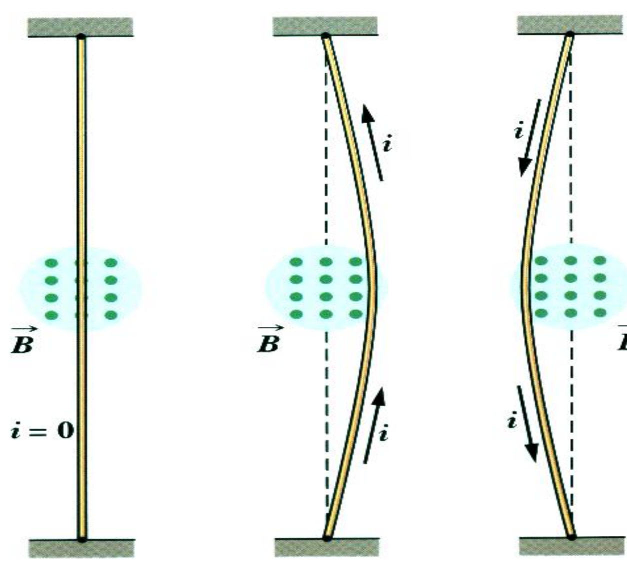
$$n = \frac{iBd}{eVA} = \frac{iB}{eVL}$$

End View



Magnetic Force on a Current-Carrying Wire

Placing a current-carrying wire within a uniform magnetic field, each charge carrier is subject to the force $\vec{F} = q(\vec{v}_d \times \vec{B})$



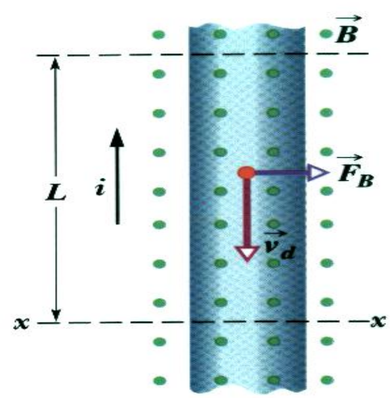


Figure 29-17

Using the total charge in a segment of length **L**, $q = it = \dfrac{iL}{v_d}$ the magnetic force is:

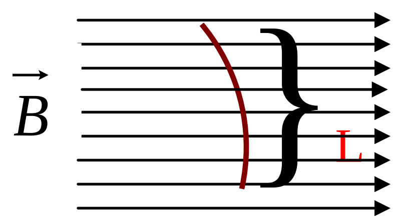
$$F = \frac{iL}{v_d} v_d B Sin(90) = BiL \qquad For \quad \vec{B} \perp \vec{v_d}$$

$$\vec{F} = i\vec{L} \times \vec{B} = BiLSin(\phi)$$

If the wire curves within the magnetic field, integration along infinitesimal length elements to yield the total magnetic force is:

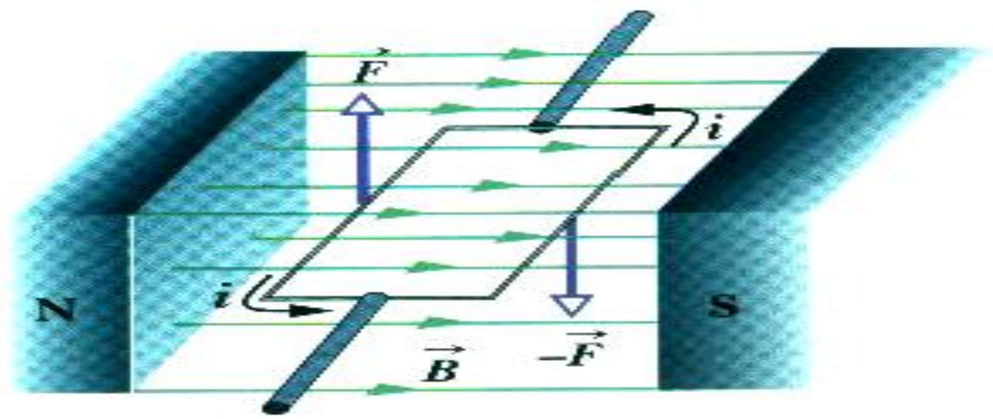
$$\overrightarrow{dF} = i\overrightarrow{dL} \times \vec{B}$$

$$\vec{F} = \int_0^L i\overrightarrow{dL} \times \vec{B}$$

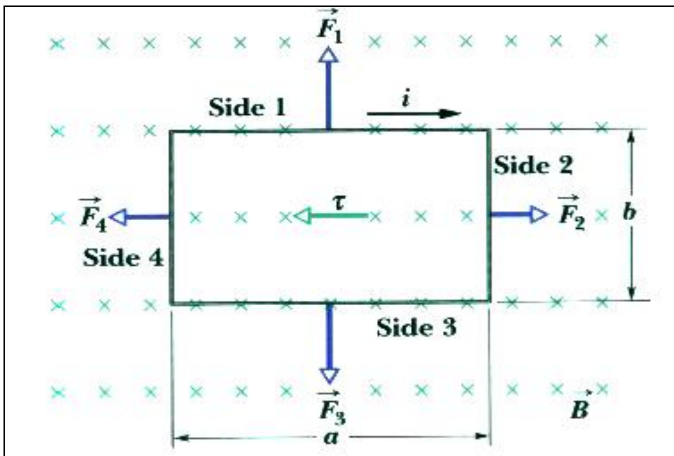


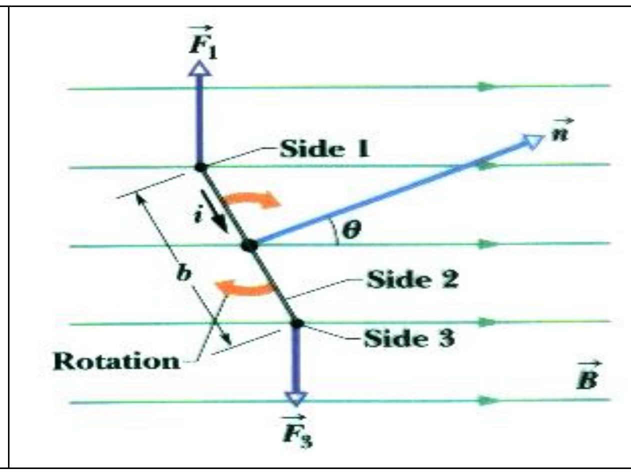
Torque on a Current Loop

Moving towards implementation of an **electric motor** (electric→mechanical), current is setup in a wire loop or a tightly wound coil of wire and placed in a $\vec{B}$ as shown.



Viewing the loop from the north pole piece, and edge-on, the loop experiences a torque that rotates $\hat{n}$ into alignment with $\vec{B}$





Torque about the central axis of rotation is: $\tau = \left\{ \frac{b}{2} F_1 + \frac{b}{2} F_3 \right\} Sin(\theta)$

Using $F_1 = Bia \quad and \quad F_3 = Bia$

$$\tau = BiabSin(\theta) = BiASin(\theta)$$

For N tightly wound loops of wire as in a coil, the total torque is:

$$\tau_N = N\tau_1 = N * iABSin(\theta)$$

For the electric motor application, as $\hat{n}$ moves into alignment with $\vec{B}$ the current direction is reversed which reverses the forces along the horizontal lengths **a** and maintains the loop rotation.

As current flows through the motor wiring loops and the loop rotates, the changing magnetic field flux through the loop generates a '**back-emf**' by Faraday induction.

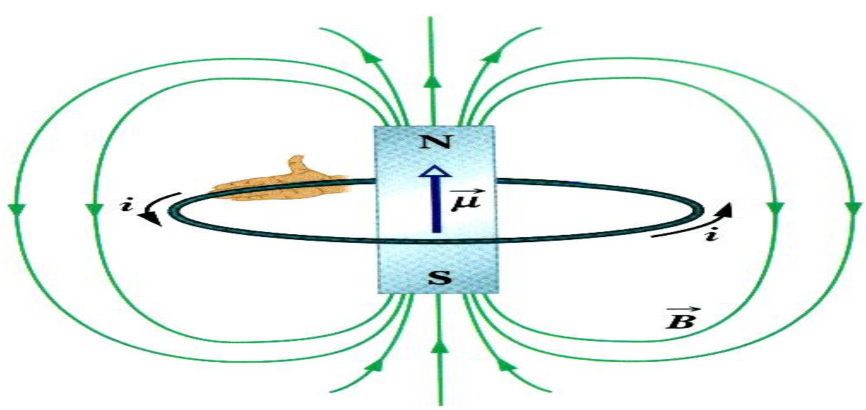
For an electric generator this induction emf is exactly what is needed, but in the electric motor, the back-emf is inefficiency and always reduces current in the coil.

Magnetic Dipole Moment

Defining the **magnetic dipole moment** of the coil as $\vec{\mu} = NiA\hat{n}$

Torque on the loop is $\tau = \vec{\mu} \times \vec{B}$

Current loops are also established by **quantum spins** of the electron, proton, or neutron, giving these objects **magnetic moments**.



In an external magnetic field, a magnetic dipole moment behaves in a similar fashion to an electric dipole moment in an external electric field. A magnetic moment vector $\vec{\mu}$ orients itself in a position of minimal potential energy collinear with the $\vec{B}$ direction.

$$U = -\vec{\mu} \cdot \vec{B}$$

Flipping a magnetic moment in an external $\vec{B}$ from a parallel to anti-parallel orientation requires an external agent do work:

$$W = +\Delta U = U_f - U_i = -\mu B Cos(180) - [-\mu B Cos(0)]$$

$$W = +2\mu B$$