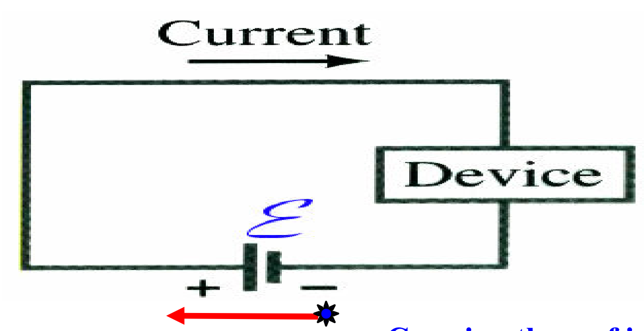
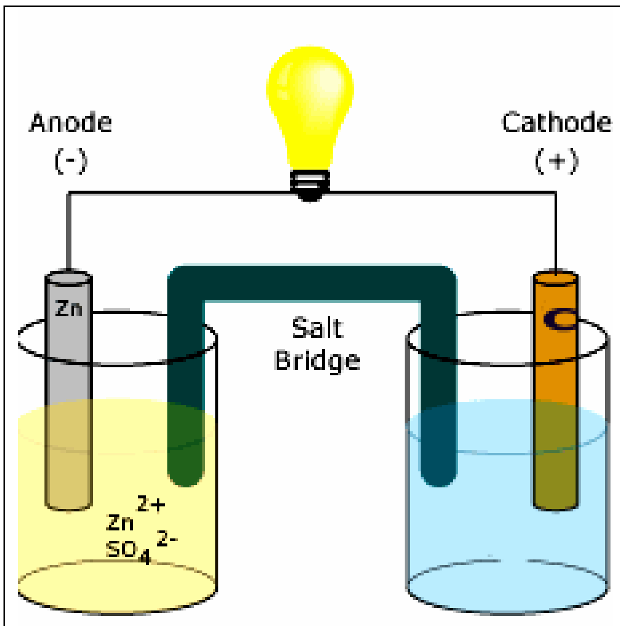
DC Circuits

Delivering a steady flow of electric charge to a circuit requires an **emf** device such as a battery, solar cell or electric generator for example. **Emf** stands for **electromotive force**, but an emf device transforms one type of energy into electric potential energy thereby establishing a $\pm$ charge separation and providing steady currents for the circuit.



Crossing the emf in this direction → $+\Delta V$

The battery was invented by **Alessandro Volta** in 1800. The fundamental component is an **electric cell** consisting of two metallic electrodes submersed in a dilute acid solution, converting chemical energy into electric energy:



Dilute sulfuric acid removes Zinc ions from the Zn strip leaving the anode negatively charged.

Electrons are drawn away from the Carbon strip leaving a positively charged C cathode.

The process continues until the Zn strip is consumed, the connection is broken or the acid is neutralized.

Without a connection between the two electrodes, the cell reaction continues until equilibrium is reached at the cell potential.

Emf is the work done per unit charge that the emf device does in moving charge from the lower potential terminal to its higher potential terminal.

$$Emf = \frac{dW}{dq} \quad in \quad volts$$

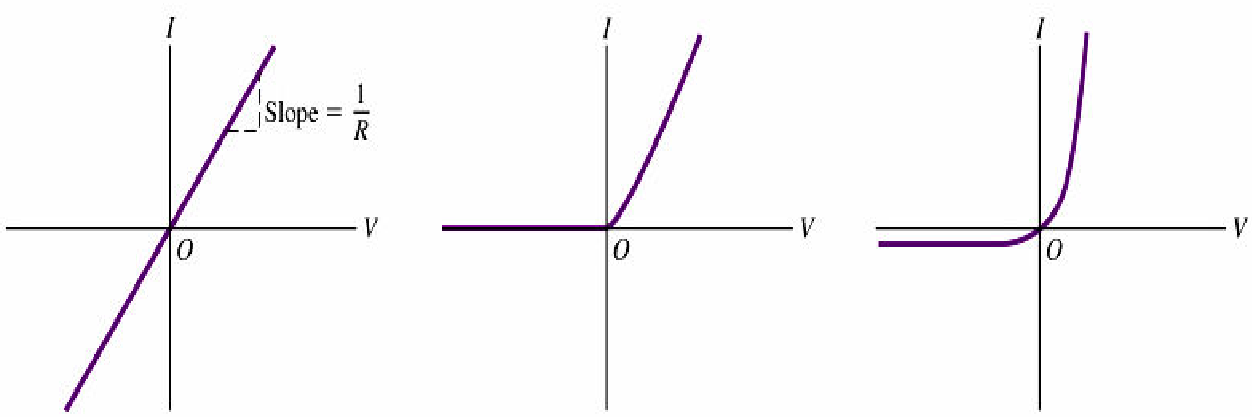
Ideally, a source of emf has zero **internal resistance 'r'**, but in reality, an internal resistance will always be present.

The "**emf**" is the **terminal potential** difference when no current flows in the device.

Resistors in Circuits

As charge carriers in a circuit encounter a 'device', motion through the device is impeded by collisions with atoms of the particular device material. Electric potential energy is converted into **thermal energy** and emf energy is transferred to the resistor.

Previously we have seen that resistors are circuit elements obeying **Ohm's law**.



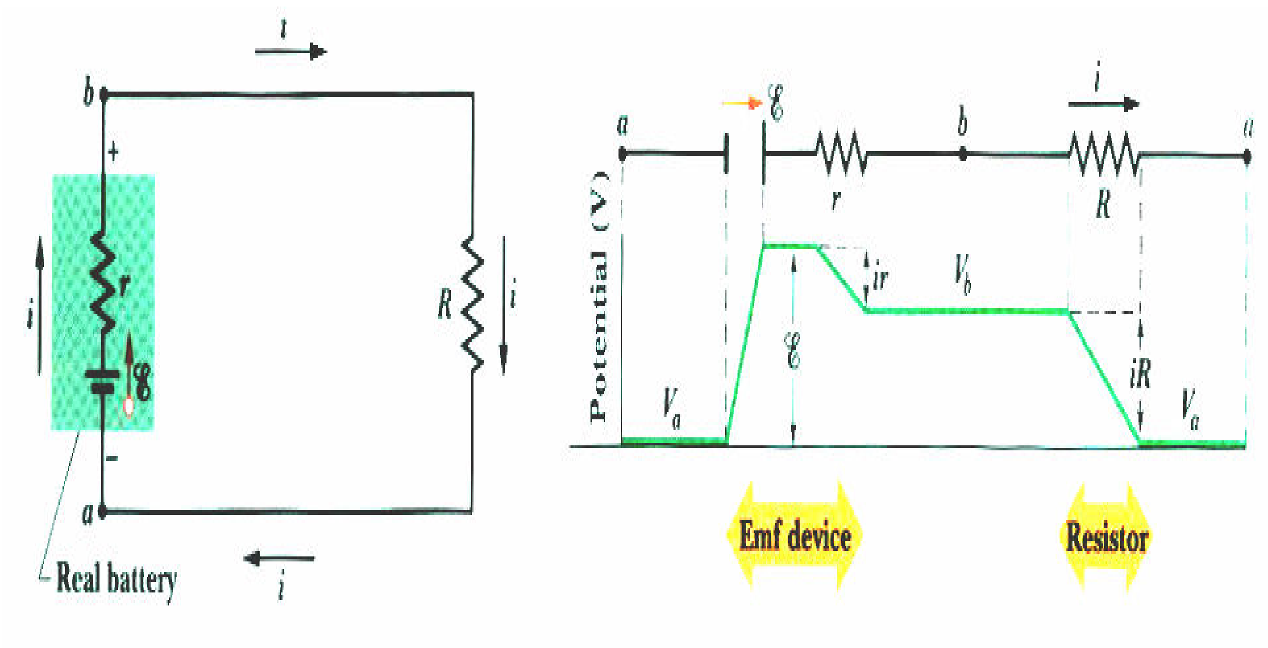
(a) Resistor that obeys Ohm's law

(b) Vacuum diode (does not obey Ohm's law)

(c) Semiconductor diode (does not obey Ohm's law)

The linear relationship $V = IR$ means charge carrier potential drops as a circuit resistor is **crossed in the direction of the conventional current** is $\Delta V = -IR$

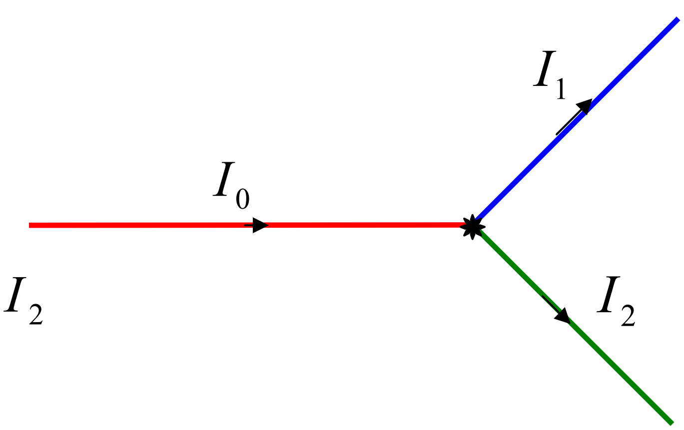
Conversely, if we evaluate the change in potential across a circuit resistor when looking in a **direction opposite to the conventional current**, we have $\Delta V = +IR$



Kirchhoff's Rules: Current Junctions & Voltage Loops

Kirchhoff's 1st Rule for circuits is a statement of the **conservation of electric charge** at a convergence point or junction within a circuit.

At any junction point, the sum of all currents into that junction equals the sum of all currents leaving the junction.

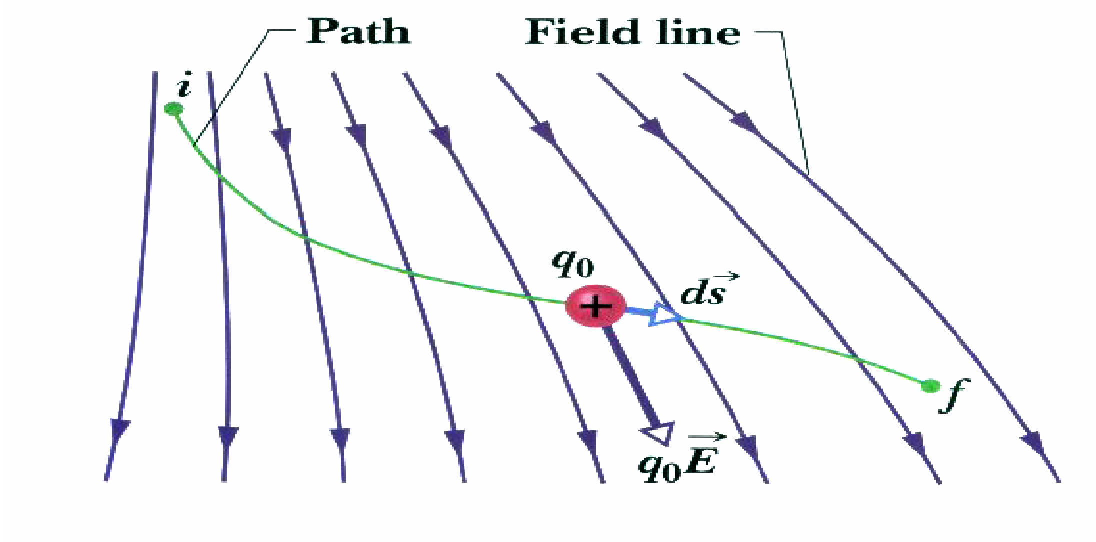


$$I_0 = I_1 + I_2$$

Kirchhoff's 2nd Rule for circuits is a statement about the change in a charge's electric potential as it moves in a closed loop path such as a circuit.

Recall the path integral $\Delta V = -\int_i^f \vec{E} \cdot \vec{dr}$ where as charge is displaced within an electric field from initial to final position, the work done per unit charge is

$$\Delta V = V_f - V_i$$

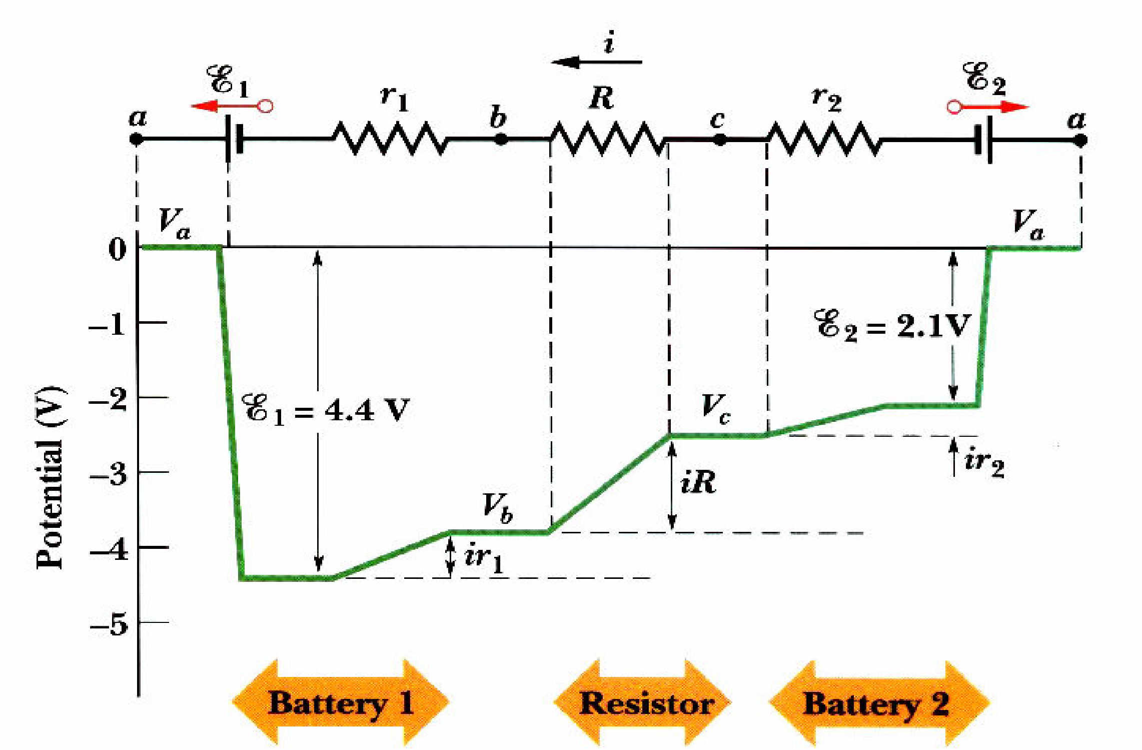


Since the electrostatic force is conservative, ΔV is independent of the path taken from start to finish and only depends on the electric potential at the end points.

In particular for the closed path, $\Delta V = \oint \vec{E} \cdot \vec{dr} = 0$

For circuits, when moving through a **closed loop** in any circuit the discrete sum of potential variations as each circuit element in the loop is crossed will be zero.

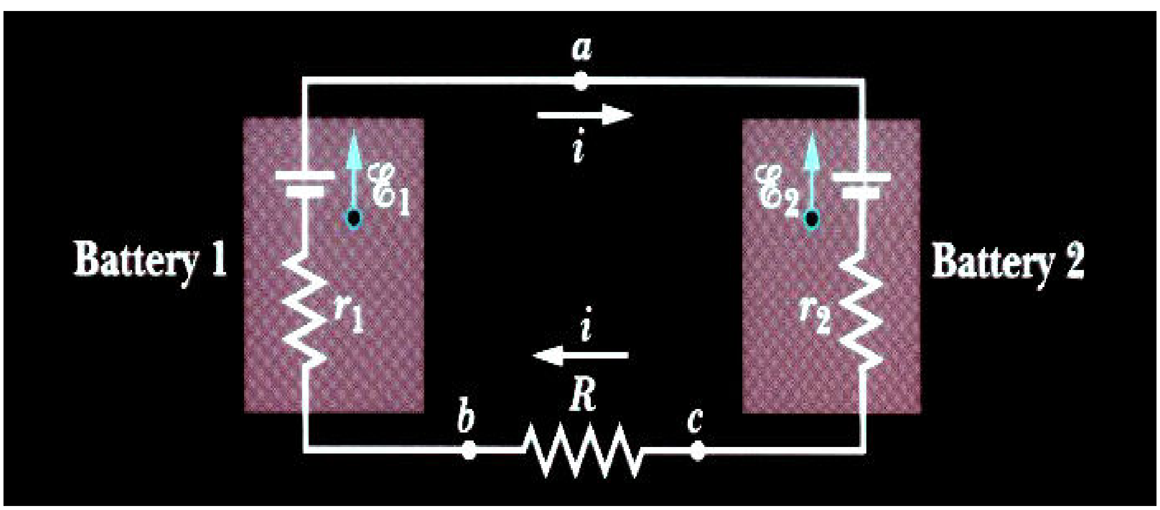
This is Kirchhoff's 2nd Rule. $\sum_i \Delta V_i = 0$



E.g., EMF with internal resistance:

In the circuit shown, find the potential difference between the terminals of battery #1.

$$E_1 = 4.4V \quad E_2 = 2.1V \quad r_1 = 2.3\Omega \quad r_2 = 1.8\Omega \quad R = 5.5\Omega$$



Write down Kirchhoff's Voltage Rule starting and finishing at point **'a'**

$$\sum_i \Delta V_i = 0$$

$$-E_2 - ir_2 - iR - ir_1 + E_1 = 0$$

Solving for the current then

$$i = \frac{E_1 - E_2}{R + r_1 + r_2} = \frac{4.4V - 2.1V}{5.5\Omega + 2.3\Omega + 1.8\Omega} = 0.240\,A = 240mA$$

The Potential difference across battery1 is

$$\Delta V = -ir_1 + E_1 = -240mA(2.3\Omega) + 4.4V = 3.8V$$

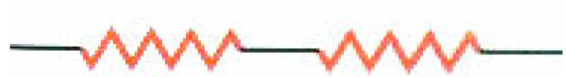
Notice this is <u>not</u> the EMF of Battery#1 which is 4.4V.

<u>Parallel and Series Resistor Networks</u>

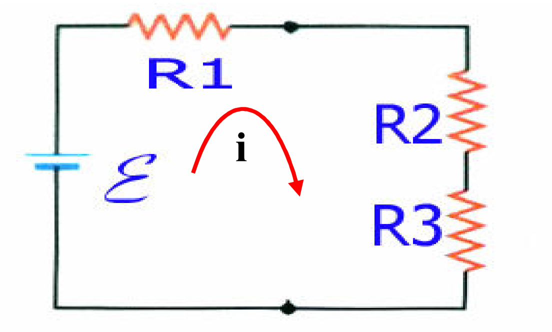
As was done with capacitors, it may be possible to reduce resistor networks into an equivalent resistance for the circuit by appropriately adding resistor combinations.

In terms of resistivity, resistance is $R = \rho \dfrac{L}{A}$

Here **<u>L</u>** is the length of the resistive material such that if we put two of these back-to-back, i.e., a **series** combination, then **<u>L</u>** doubles and the resistance doubles.



Formally, we can also find this by considering the following circuit:



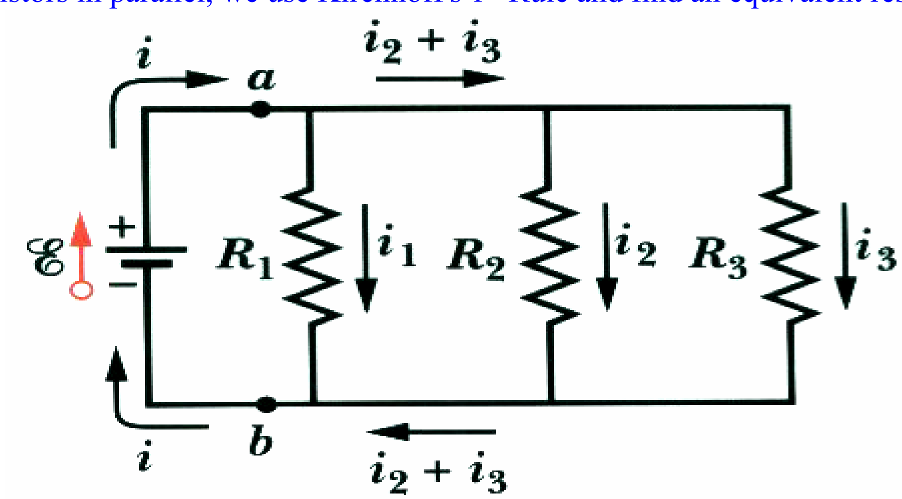
$$\sum_i \Delta V_i = 0$$

$$E - i(R1 + R2 + R3) = 0$$

$$R_{Equivalent} = R1 + R2 + R3$$

<u>**Or, for resistors in series,**</u> $$R_{Equivalent} = \sum_i R_i$$

For resistors in parallel, we use Kirchhoff's 1st Rule and find an equivalent resistance as:



We know from Ohm's Law that: $i_1 = \frac{E}{R_1} \qquad i_2 = \frac{E}{R_2} \qquad i_3 = \frac{E}{R_3}$

In addition, we want to write something like: $E - iR_{Equivalent} = 0$

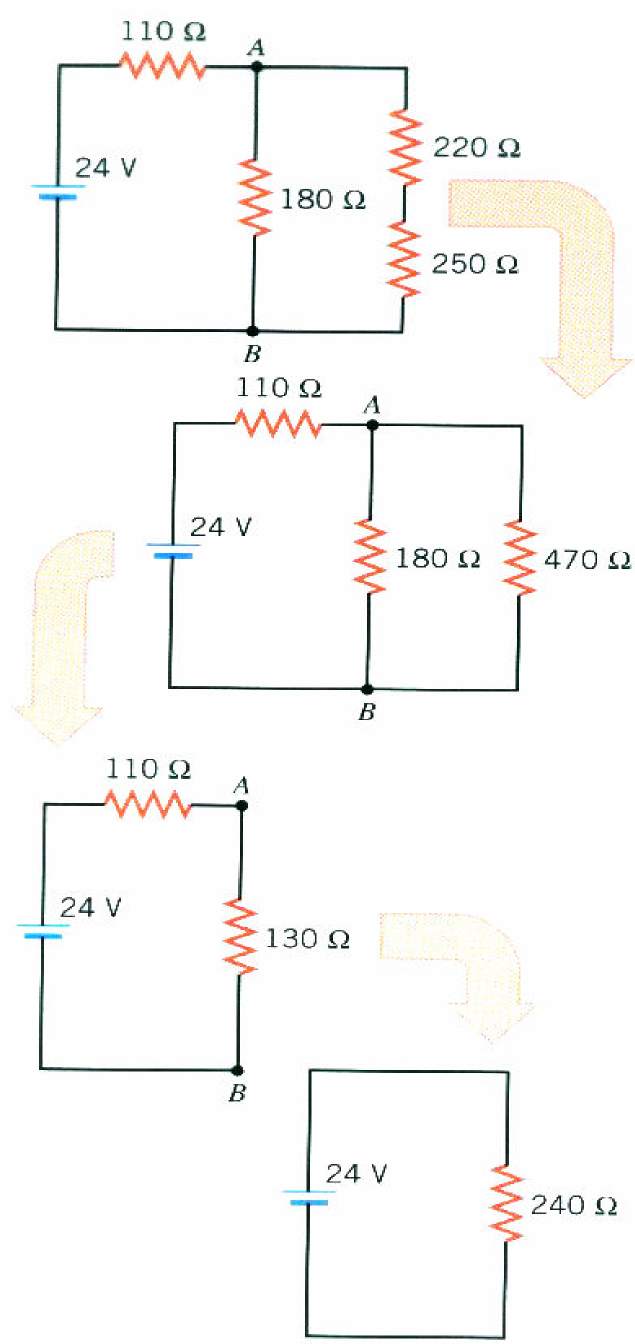
From Kirchhoff's 1st Rule at junction **a**, $i = i_1 + i_2 + i_3$

$$i = E\left\{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right\} = E * \frac{1}{R_{Equivalent}}$$

$$R_{Equivalent} = \left\{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right\}^{-1}$$

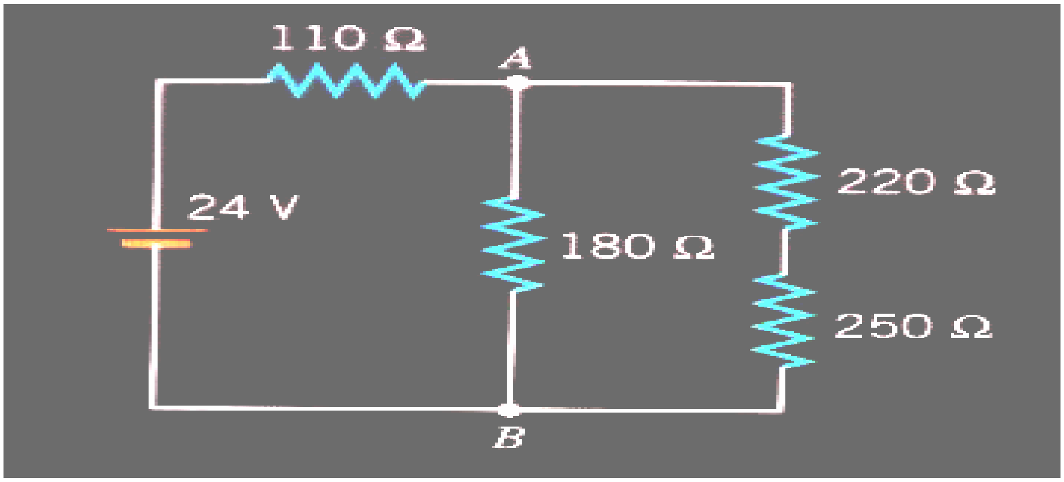
In general for resistors in parallel, $\frac{1}{R_{Equivalent}} = \sum_i \frac{1}{R_i}$

E.g., Resistor Network Reduction:



E.g., Current in a Resistor Network:

Find the current in the battery and through the 250Ω resistor for the circuit shown below.



$$i = \frac{V}{R_{Equivalent}} = \frac{24V}{110\Omega + 180\Omega // 470\Omega} = \frac{24V}{110\Omega + \left(\frac{1}{180\Omega} + \frac{1}{470\Omega}\right)^{-1}}$$

$$i = \frac{V}{R_{Equivalent}} = \frac{24V}{240\Omega} = 0.1A = 100mA$$

This is current through the battery and the equivalent circuit

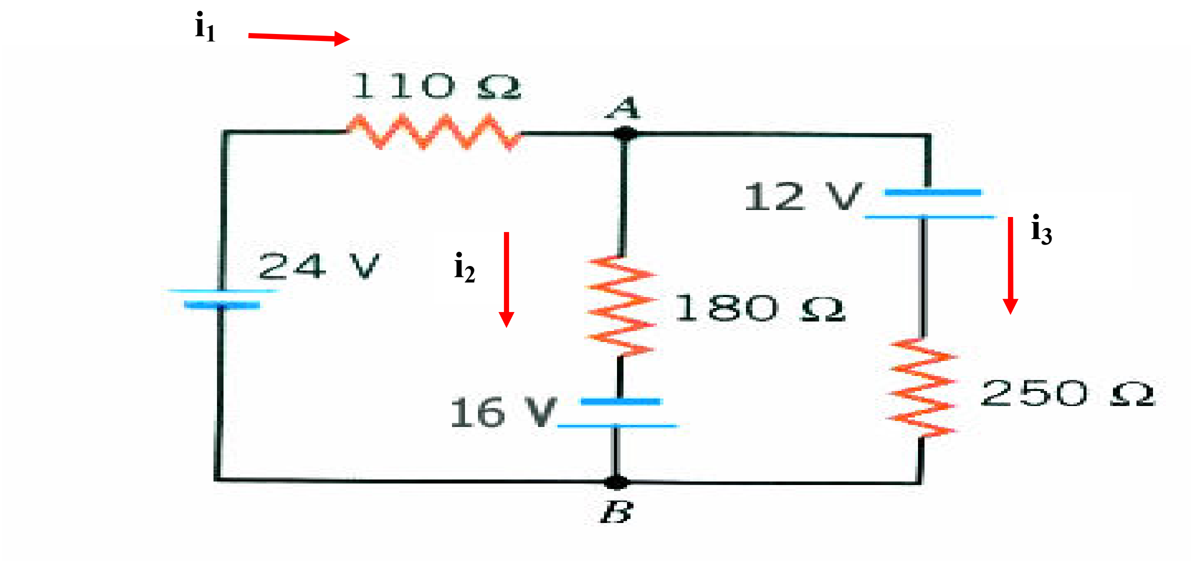
For the current in the 250Ω resistor, note that the voltage at point **A** is

$V_A = 24V - i(110\Omega) = 13V$ Which is also the voltage on the parallel connection of the 180Ω resistor with the 250Ω, 220Ω combination.

The current in the 250Ω, 220Ω combination is therefore

$$i_{250\Omega\_220\Omega} = \frac{13V}{470\Omega} = 27.7mA$$

E.g., Multi-loop Network: Find the currents using Kirchhoff's Rules.



At Junction A; $i_1 = i_2 + i_3$

Clockwise around the 'left' loop;

$$24V - i_1(110\Omega) - i_2(180\Omega) + 16V = 0$$

Clockwise around the 'right' loop;

$$12V - i_3(250\Omega) - 16V + i_2(180\Omega) = 0$$

Leaving us three equations with three unknowns i_1, i_2, i_3

From the first and second equations:

$$24V - (i_2 + i_3) * (110\Omega) - i_2(180\Omega) + 16V = 0$$

From this equation and the 'right' loop equation:

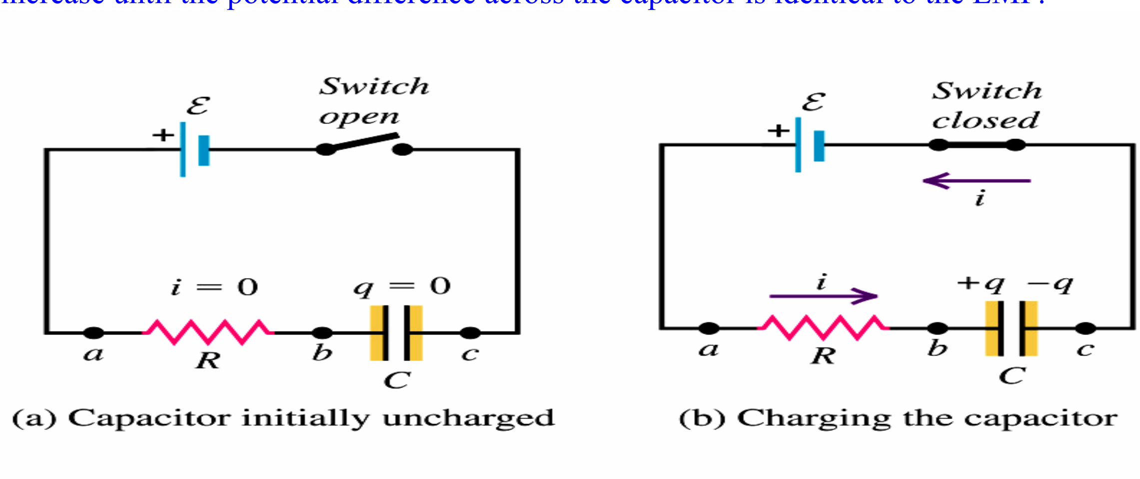
$$24V - (i_2 + \frac{-4V + i_2(180\Omega)}{250\Omega}) * (110\Omega) - i_2(180\Omega) + 16V = 0$$

Resulting in: $i_2 = 113mA \quad i_3 = 65.4mA \quad i_1 = 178mA$

RC Series Circuits

The DC charging / discharging characteristics of series RC circuits may be determined using Kirchhoff's voltage loop rule:

In the **charging case**, as the switch is closed, the charge on the capacitor continues to increase until the potential difference across the capacitor is identical to the EMF.



Kirchhoff's Voltage Rule gives:

$$E - \frac{q}{C} - iR = 0 \quad \rightarrow \quad \frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$$

The homogeneous equation has $\frac{dq}{dt} + \frac{q}{RC} = 0$

$$\frac{dq}{q} = -\frac{dt}{RC} \quad \rightarrow \quad \ln|q| = -\frac{t}{RC} + k$$

$$q(t) = ke^{-t/RC}$$

The particular equation has $q(t) = CE$ as its solution so a general solution is:

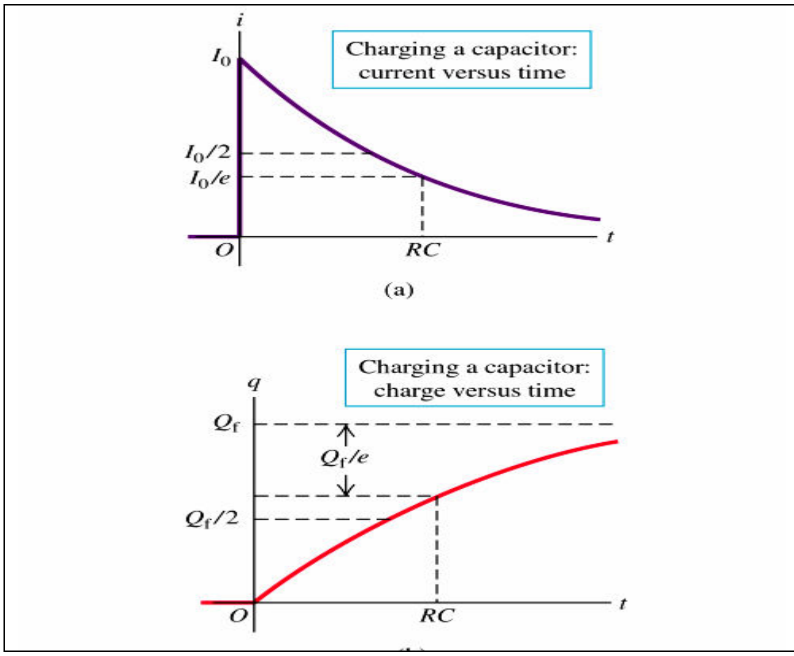
$$q(t) = CE(1 - e^{-t/RC}) = CE(1 - e^{-t/\tau})$$

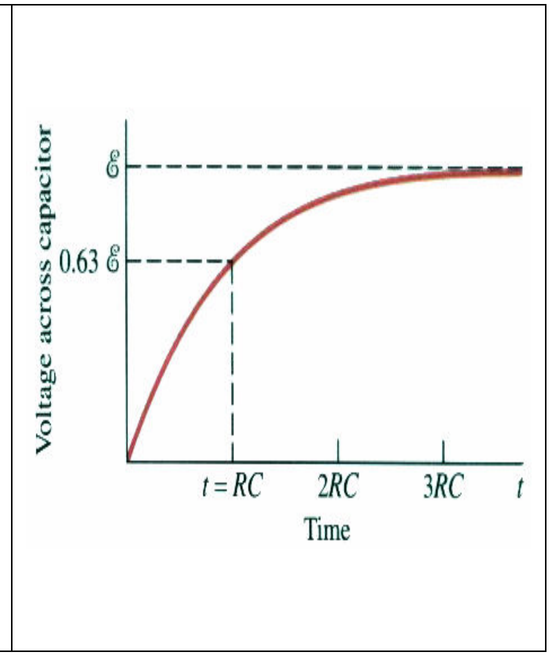
$$\tau = RC = Capacitive\_Time\_Const.$$

The current in the circuit may be found by considering $i(t) = \frac{dq}{dt} = \frac{E}{R}e^{-t/\tau}$

Note the capacitor becomes an open as time goes to ∞.

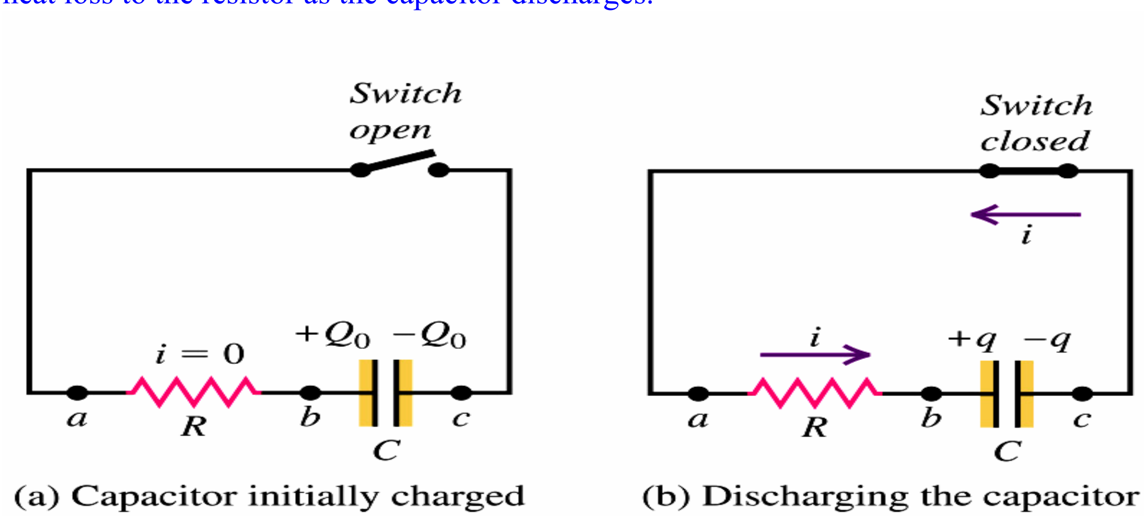
The voltage on the capacitor as it charges is $V(t) = \frac{q(t)}{C} = E(1 - e^{-t/\tau})$





<u>Discharging case</u>:

The capacitor starts out initially with a voltage of $V_0 = E$ and current is dissipated as heat loss to the resistor as the capacitor discharges.



(a) Capacitor initially charged (b) Discharging the capacitor

$$-\frac{q}{C} - iR = 0$$

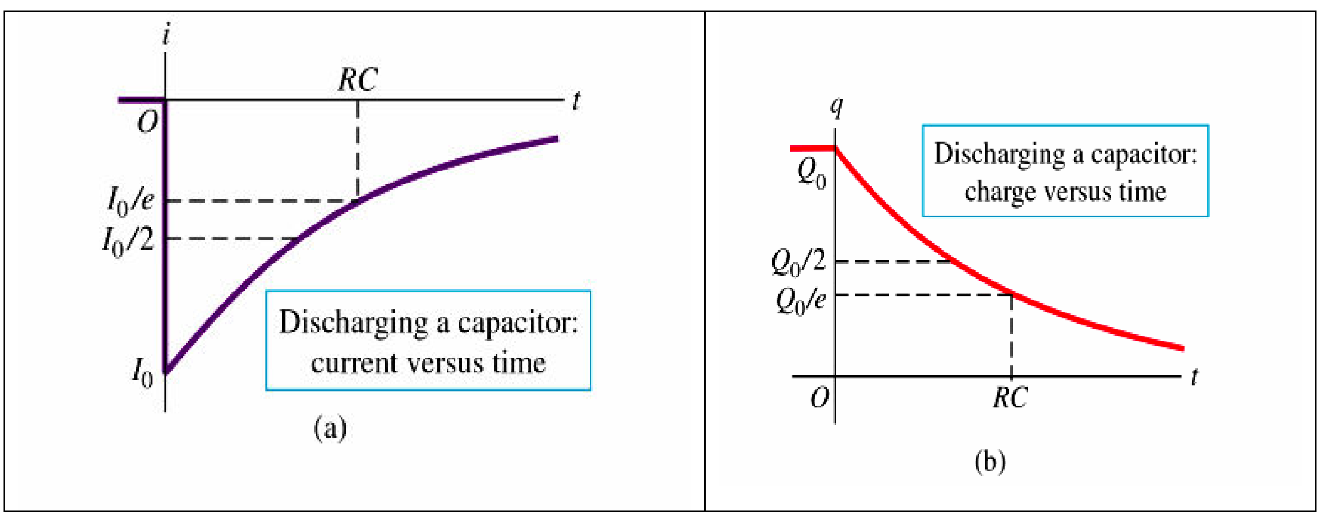
$$\frac{dq}{dt} + \frac{q}{RC} = 0$$

$$q(t) = ke^{-t/RC} \qquad q(0) = q_0 \Rightarrow k = CV_0$$

$$q(t) = CV_0 e^{-t/RC} = CV_0 e^{-t/\tau}$$

$$i(t) = \frac{dq}{dt} = -\frac{V_0}{R} e^{-t/\tau}$$

$$V(t) = \frac{q(t)}{C} = V_0 e^{-t/\tau}$$



At **$t = 1\tau$, $V \sim 0.37V_0$** At **$t = 5\tau$, $V \sim 0.01V_0$**

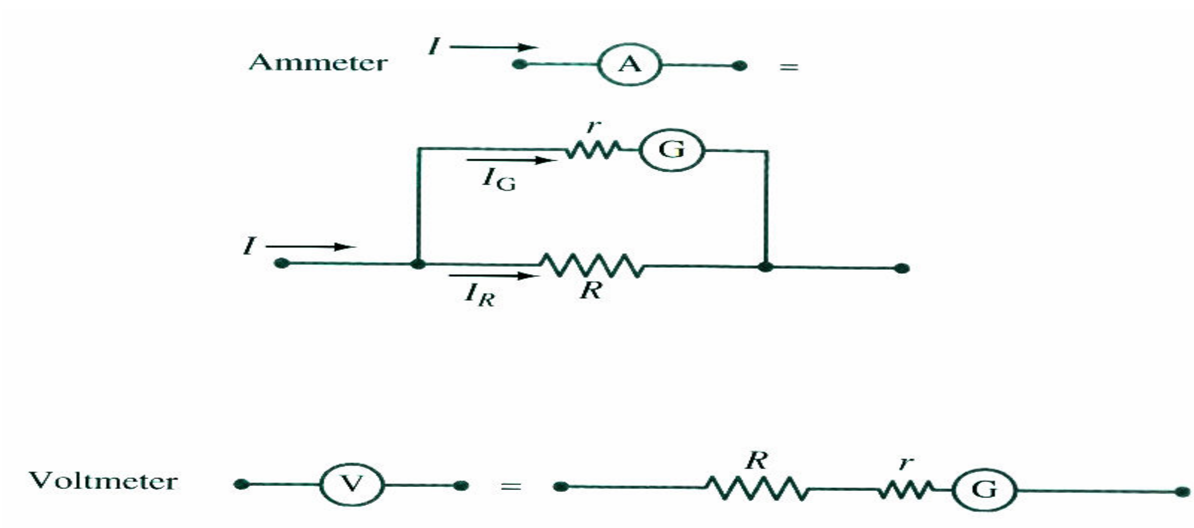
Ammeters, Voltmeters and Ohmmeters.

Making measurements of current, voltage and resistance within a circuit requires different techniques depending on the measured quantity.

In the ammeter, R is small such that little to no potential drop occurs across R.

In the voltmeter, R is large such that little to no current is drawn through R.

In both cases shown, **G** is a galvanometer with a coil of wire placed inside a magnetic field to gauge the current, voltage or resistance through / across a wire or element. A current-carrying coil experiences a torque in the magnetic field which, when calibrated, rotates an analog meter needle to the readout value.



Note the hook-up: **in-line for the ammeter** or **across the element** for the voltage and resistance measurements.

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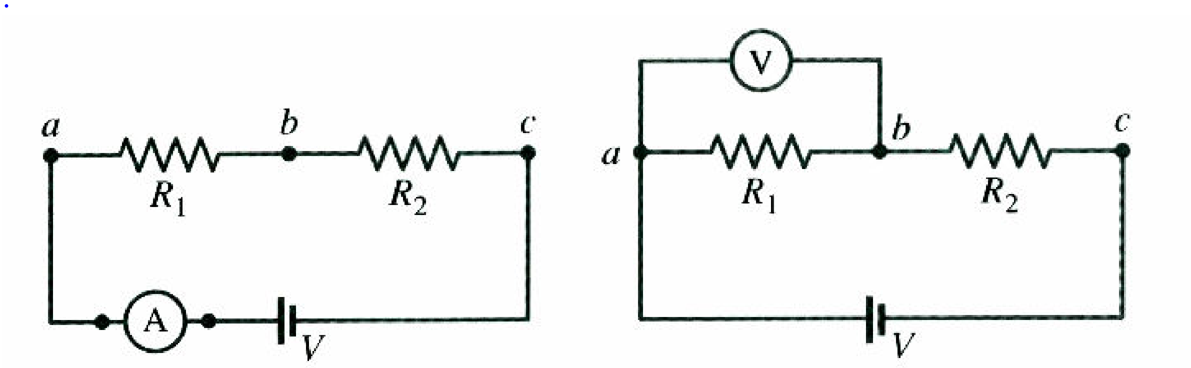
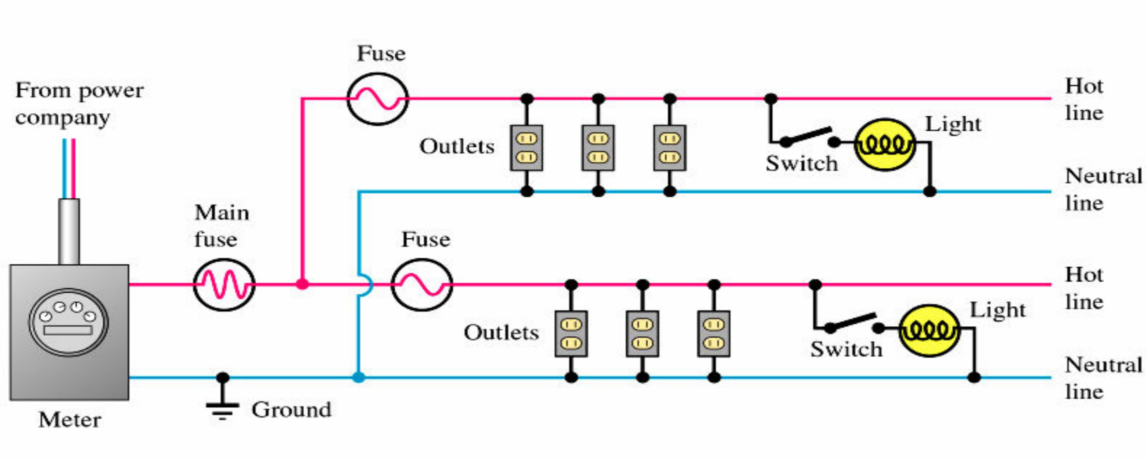


Power Distribution

Electric power supplied from power plants is generated mainly by burning coal, oil, or gas, nuclear reactions, and hydroelectric. Electricity is **alternating current (AC)** 120V and fused within the home. **Shorts**, surges, device malfunction or other circumstances that create excessive current within the wiring are protected against with a fuse or circuit breaker designed to create an **open** in the circuit when currents are unsafe.



Fuse sizes are determined by the amount of resistive heating the wiring can safely handle. This in turn depends on wire gauges such that upping a fuse arbitrarily is not a good idea.

12-Gauge Copper Wire	**2.05-mm Diameter-20A max-Lighting**
8-Gauge Copper Wire	**3.26-mm Diameter-Appliances**
6-Gauge Copper Wire	**4.11-mm Diameter-Appliances**
2-Gauge Copper Wire	**6.54-mm Diameter-Main Power Lines**

AC current / voltage transmissions have the advantage of not being as costly in terms of dissipative resistive loss. Since power is $P = IV = I^2R$ the average AC power includes averaging over a Sine squared function which gives a factor of $\frac{1}{2}$

$$V = V_0 Sin(\omega t)$$

$$I = I_0 Sin(\omega t)$$

Where $\omega / 2\pi = 60Hz$

$V_0 \quad and \quad I_0$ **Peak values** of the voltage and current.

$$\overline{P} = RI_0^{\ 2}\overline{Sin^2 \omega t} = \frac{1}{2} I_0^{\ 2} R$$

Measured at the wall with a **DVM** are the **RMS** voltage and current:

$$V_{rms} = \sqrt{\overline{V^2}} \qquad I_{rms} = \sqrt{\overline{I^2}}$$

$$\overline{P} = I_{rms} V_{rms} = \frac{1}{2} I_0 V_0$$

➔

$$I_{rms} = {I_0}/{\sqrt{2}} \qquad V_{rms} = {V_0}/{\sqrt{2}}$$

Further, AC transmission at high voltage (~100's kV) and low currents, further reducing the resistive loss, is possible since transforming high voltages with step-down transformers for consumer use is easier with AC.