Electromagnetism

Electromagnetism unifies electric and magnetic phenomena into a common theoretical framework regarding each as manifestations of the same underlying fundamental electromagnetic force.

A qualitative connection between electricity and magnetism was made in 1819 when **Hans Christian Oersted** and **André Ampere** observed electric currents deflecting nearby compass needles indicating the presence of a magnetic field.

In the 1830's, **Michael Faraday** and **Joseph Henry** showed experimentally that it was possible to induce electric currents in a circuit by changing magnetic field configurations.

Using the empirical results of Oersted, Ampere, Faraday and Henry, the quantitative connections linking electric and magnetic phenomena known as **Maxwell's Equations** were deduced in 1864 by **James Clerk Maxwell**.

Notably, Maxwell's equations lead to wave equations for electric and magnetic fields thus predicting the existence of propagating **electromagnetic waves** (waves consisting of oscillating electric and magnetic fields) that require no medium in which to propagate and which can transport energy at the speed of light. **Light** is an electromagnetic wave.

Electromagnetism is approached on four fronts:

1) **Electrostatics**
2) **Magnetostatics**
3) **Electrodynamics**
4) **Magnetodynamics**

The '**-statics**' implies charges are stationary and electric / magnetic fields are not changing in time.

The '**-dynamics**' means that electric / magnetic fields are changing in time

Results from each of these four areas are described separately and Maxwell's equations with electromagnetic waves complete the discussion toward semesters end.

Electrostatics

The stationary charges of interest in electrostatics are **electric charges**.

Electric charge is an **intrinsic property** of fundamental particles like the electron, quarks, etc., which causes these particles to interact through the electromagnetic force.

Benjamin Franklin (1700's) determined the two flavors of electric charge, positive and negative, exhibiting the property that two **unlike** charges produce an electromagnetic attraction between the charges and two **like** charges will repel each other.

In 1785 **Charles Augustin Coulomb** quantified the electric force of attraction / repulsion by stating **Coulomb's Law** which holds true for static point charges:

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

This is a vector equation with the force direction along a line joining the two point charges.

Superposition holds and a vector addition of individual Coulomb forces gives the net force if more than two charges are involved.

q_i are the values of the electric charges in SI units of **Coulombs** C.

1 Coulomb = Amount of charge from 1 Amp in 1s. Approximately $6x10^{18}$ electrons.

r^2 is the distance between charges squared.

k is a constant and in SI units its value is $8.99x10^9\, Nm^2/C^2$

$k = \dfrac{1}{4\pi\varepsilon_0}$ Where $\varepsilon_0 = 8.85x10^{-12}\dfrac{C^2}{Nm^2}$ is the **permitivity of free space**

Shell theorems resembling those from Universal Gravitation are:

1) **A test charge outside a uniformly charged conducting shell experiences a net electrostatic force as if all the shell's charge were located at its center point**

2) **A test charge located within a uniformly charged conducting shell experiences a zero net electrostatic force from the shell.**

Properties of electric charge:

1) In 1917 **Robert Millikan** discovered that **electric charge is quantized** in discrete units of $e = 1.602x10^{-19} C$ Charged objects may carry **1e, 2e…43e,** but never non-integer multiples like **2.73e** or **6.39e** of electric charge**.**

2) **Electric charge is conserved.** Neither created nor destroyed. Negatively charged electrons transferred away from an **electrically neutral** object results in that object acquiring a net positive charge. The net charge before the electron transfer was zero and afterwards the number of negative charges removed is offset by an equal quantity of positive charge on the originally neutral object.

3) Atomic structure leads to the property that **bulk material is electrically neutral.** There are usually as many nuclear protons (charge +1e) as there are electrons (charge -1e) in the electronic **'orbitals'.**

4) Electrons are more easily removed from an atom as compared to the protons. **(α~0.01 , α_s~1).** Charge transfers away from or onto an object are usually in the form of electrons and not protons, which are more tightly bound in the nucleus**.**

5) **Electric conductivity** is a measure of how mobile electrons are within a material**.**

   a) **Conductors** are materials where electrons move freely. Metals typically**.**

   b) **Electrically insulating** materials (glass, wood, rubber…) have electrons that are more strongly bound to their nuclei that do not move easily within the material**.**

   c) **Semiconductors** have intermediate conductivity properties making these materials useful in electronics applications where precise control of electric current is critical **(Si, Ge, C).**

   d) **Superconductors** are materials that have zero resistance to electric current flow at extremely low temperatures**. (4 K to ~160 K).**

The Electroscope

The **Amber Effect** noticed by the Greeks in 600 BC, is an example of a charge transfer (order of a micro-coulomb) by rubbing, where a material like amber, glass, plastic, rubber, etc., gains a **net electric charge** by rubbing it with a cloth/fur.

In the process **electrons** either:

a) Leave the cloth/fur and are deposited onto the material giving the material a net negative charge and leaving the cloth/fur positively charged**.**

b) Are removed from the material leaving it positively charged and the cloth/fur with an equal and opposite negative charge**.**

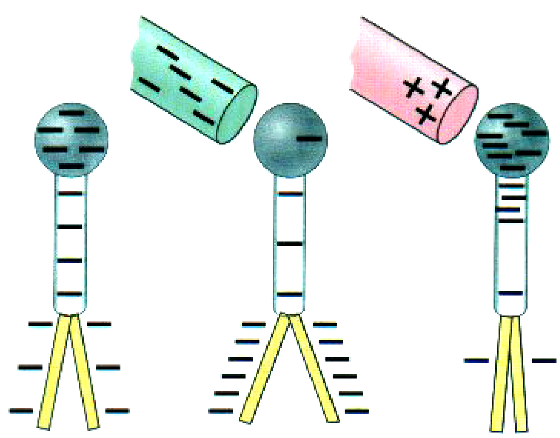
Since charge is conserved, an electrically neutral object to begin with (net charge = 0) implies a **total net charge** of zero on the cloth/fur-material system.

An excess of electric charge is a non-equilibrium condition for anything except fundamental particles.

Having charged a material by the Amber effect its charge diminishes through **leakage** or by flow to a **ground** (charge reservoir) if the object is not **electrically isolated**.

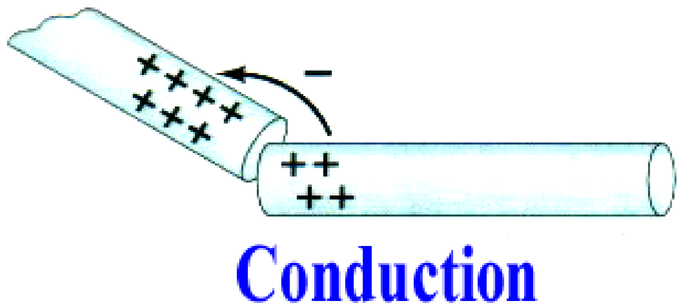
An **electroscope** can determine the presence of electric charge and its arithmetic sign:

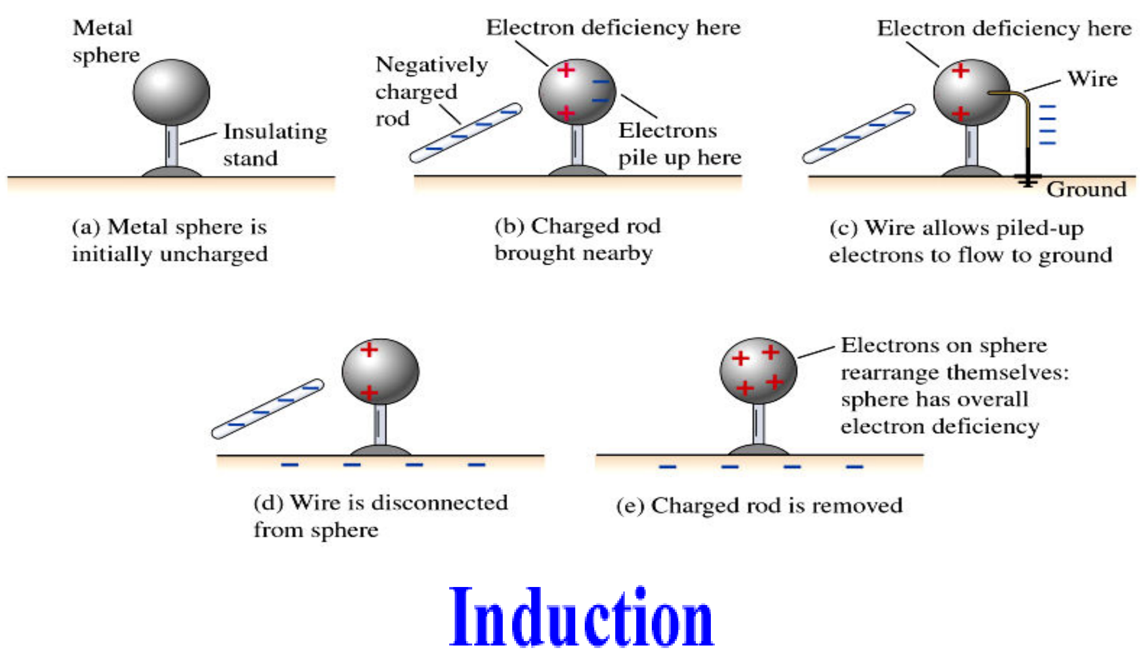
Comprised of a conducting metal sphere attached to two gold leafs free to move under the influence of Coulomb attractions / repulsions, the response of an electroscope is shown below. Arithmetic sign of the objects charge is determined by the push/pull response of the electroscope leafs to the charged object as it nears.



Conduction and Induction

Charge transferred from one object to the next by physical contact is referred to as **conduction**. Charge induced on another object when the charge-inducing object is close enough to produce a charge separation is called **induction**.





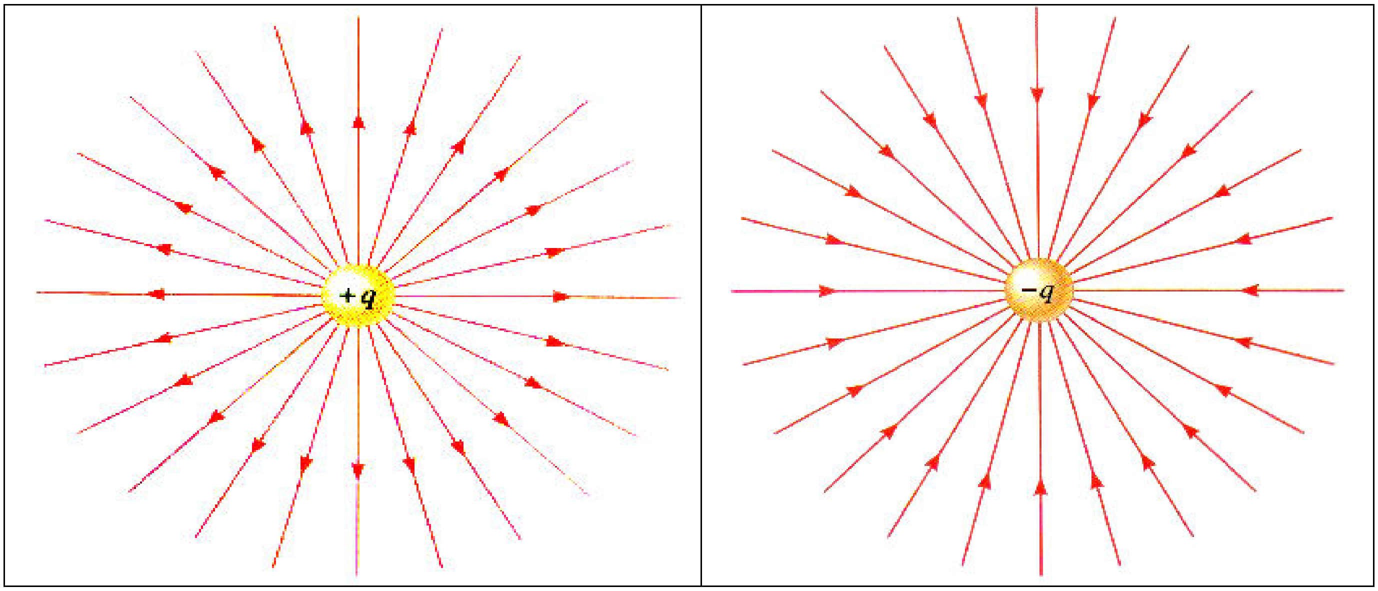
Electric Fields

The **electric field** is a concept introduced by Faraday that facilitates qualitative and quantitative understanding of electromagnetism problems.

Evaluation of the electric field at a particular point in space corresponds to determining the net force per unit charge that the existing system of charges (discrete or continuous) will exert on a **positive test charge** introduced into the environment.

The electric field $\vec{E}$ represents an intermediary between preexisting charge distributions and any charge introduced into the environment.

<u>Graphically electric field lines extend away from positive charges and into or toward negative charge with field line densities indicating relative field strength.</u>



<u>The electric field direction is tangent to the electric field lines.</u>

<u>The electric field is a vector field</u>. It is therefore necessary to specify direction and magnitude at each point in space where the field exist.

$$\vec{E}(\vec{r}) = (E_x(r)\hat{i} + E_y(r)\hat{j} + E_z(r)\hat{k})$$

Given a positive test charge q_0 experiences a force $\vec{F}$ at some location **P;**

The electric field is defined as $\vec{E} \equiv \dfrac{\vec{F}}{q_0}$ In a direction collinear with $\vec{F}$

The SI units of electric field strength are N/C

Example:

To find the electric field at point **P** due to a **point charge** q, find the force a $+$ test charge q_0 experiences at point **P** and divide by the charge on this test charge q_0:

$$|\vec{E}| = \frac{k\dfrac{|q||q_0|}{r^2}}{|q_0|} = k\frac{|q|}{r^2}$$

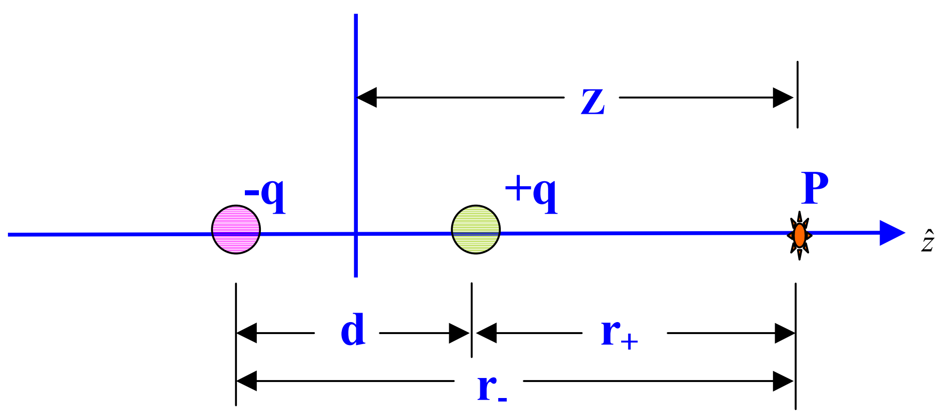
The electric field due to a point charge falls off as $1/r^2$ and is directed away from q if q is positive and is directed towards q if q is negative.

By superposition, the electric field of a discrete system on **N** charges is the vector sum of the individual electric fields:

$$\vec{E} = \frac{\vec{F}}{q_0} = \vec{E}_{Charge\_1} + \vec{E}_{Charge\_2} + \ldots + \vec{E}_{Charge\_N}$$

Example:

The on-axis electric field due to an **electric dipole** is:



$$\vec{E}(P) = \vec{E}_{+} + \vec{E}_{-}$$

$$|\vec{E}(P)| = k\left\{\frac{q}{r_{+}^{2}} - \frac{q}{r_{-}^{2}}\right\} = kq\left\{\frac{1}{(z-\frac{d}{2})^{2}} - \frac{1}{(z+\frac{d}{2})^{2}}\right\}$$

$$|\vec{E}(P)| = kq\left\{\frac{1}{z^{2}\left(1-\frac{d}{2z}\right)^{2}} - \frac{1}{z^{2}\left(1+\frac{d}{2z}\right)^{2}}\right\}$$

$$|\vec{E}(P)| = \frac{kq}{z^{2}}\left\{\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right\}$$

For **Z >>d,** a binomial expansion of the terms in parentheses is valid:

$$(1+x)^{n} = 1 + nx + n(n-1)x^{2} + \ldots$$

For the above terms, **n = -2 and x = $\pm$ d/2z**

$$\left(1-\frac{d}{2z}\right)^{-2} = 1-2\left(\frac{-d}{2z}\right) \quad \textbf{\&} \quad \left(1+\frac{d}{2z}\right)^{-2} = 1-2\left(\frac{d}{2z}\right)$$

$$|\vec{E}(P)| = \frac{kq}{z^2}\left\{\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right\} = \frac{kq}{z^2}\left(\frac{2d}{z}\right)$$

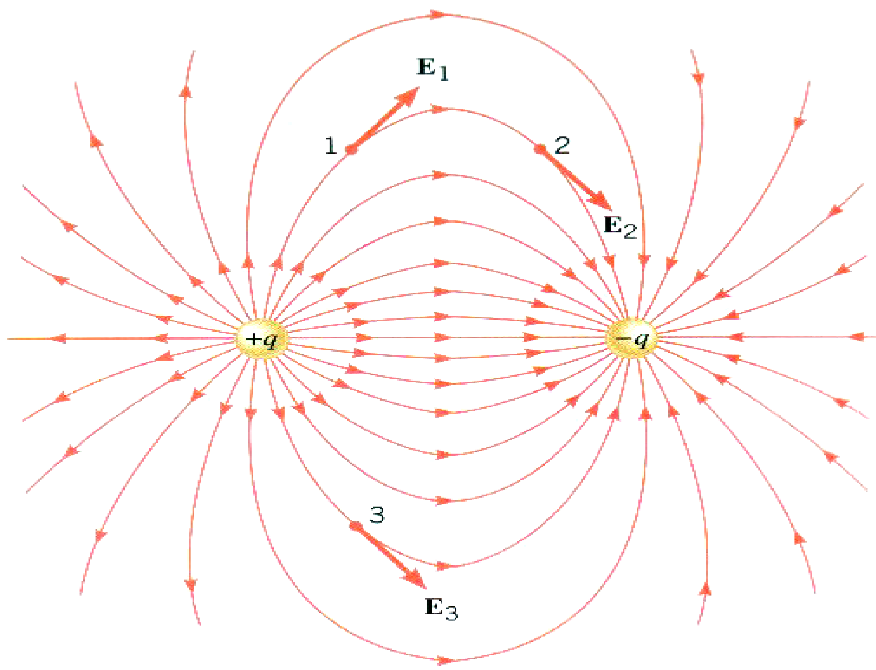
$$|\vec{E}(P)| = \frac{k2qd}{z^3}$$ **Or** $$\vec{E}(P) = \frac{1}{2\pi\varepsilon_0} * \frac{\vec{P}}{z^3}$$

The electric dipole moment vector is defined as:

$$\vec{P} = qd \qquad directed\_from\_-q\_to\_+q$$

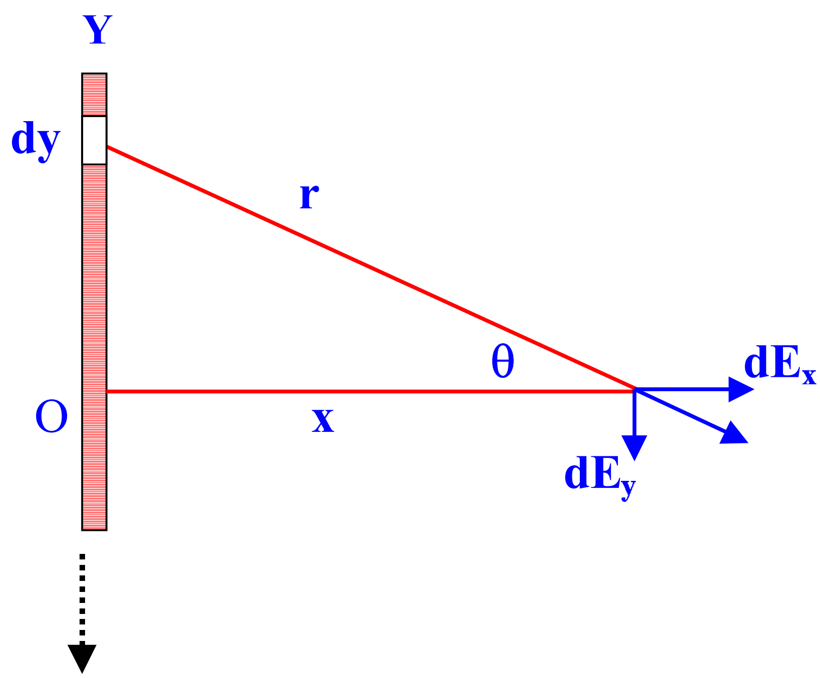
The electric field of a dipole falls off as $1/z^3$ versus the $1/r^2$ point charge result.

The electric field strength reduction results from the **+q, -q** proximity.



Example:

Electric field due to an **infinite line of charge**:



$$dE = \frac{kdQ}{r^2} = k\frac{\lambda dy}{r^2} = k\frac{\lambda dy}{x^2 + y^2}$$

λ is the **linear charge density** with SI units $[C/m]$

Since the **'Y'** components of the resulting field cancel when the entire line of charge is integrated over, the calculation of interest is:

$$E = E_x = \int dE * Cos(\theta) = k\lambda \int Cos(\theta)\frac{dy}{x^2 + y^2}$$

From the figure, y = x*Tan(θ) ➔ **dy = x*(Sec^{2}θ)dθ**

The denominator is $x^2+y^2 = x^2*(1+ \tan^2(\theta)) = x^2*(\sec^2(\theta))$

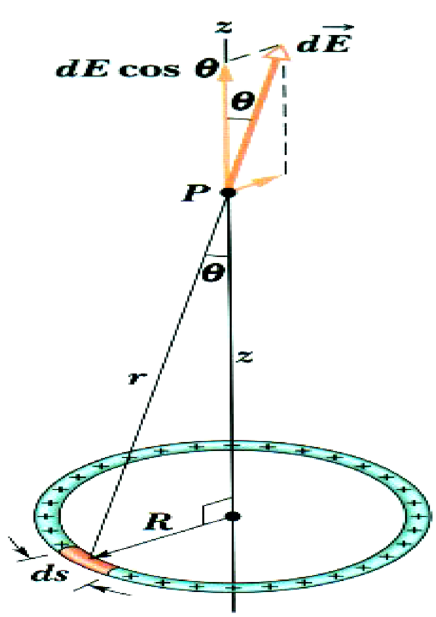
$$E = k\lambda \int Cos(\theta) \frac{dy}{x^2+y^2} = \frac{k\lambda}{x} \int_{-\pi/2}^{\pi/2} Cos(\theta) d\theta$$

$$E = \frac{2k\lambda}{x}$$

This is a good approximation also for finite lengths of charge when **X is small in comparison to the distance of P** from the wire ends.

Example:

The on-axis electric field due to a **ring of charge** with linear charge density λ.



The horizontal components of the electric field cancel by symmetry.

The differential element of charge $dq = \lambda ds$.

The calculation of interest is for electric field components parallel to the z-axis.

Field is directed away from the ring if it is positively charged and toward the ring if it is negatively charged.

$$dE_{//} = dE\,Cos(\theta) = \frac{kdQ}{r^2} Cos(\theta) = \frac{k\lambda ds}{R^2 + z^2} \frac{z}{r}$$

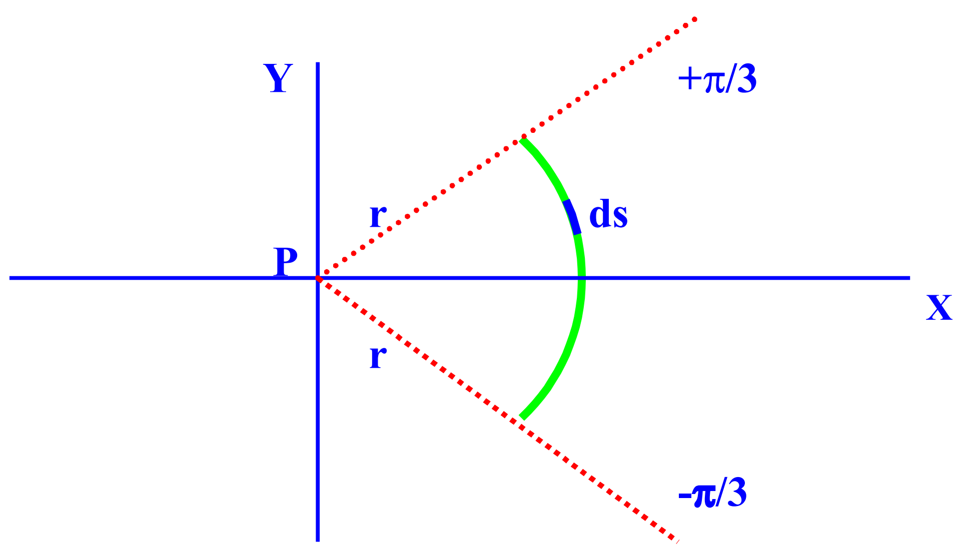
$$E_{//} = \int_0^{2\pi R} \frac{k\lambda ds}{R^2 + z^2} \frac{z}{\sqrt{R^2 + z^2}} = 2\pi R k\lambda * \frac{z}{\left(R^2 + z^2\right)^{\frac{3}{2}}}$$

Since λ = q / 2πR, $$E_{//} = \frac{kqz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}$$ **Note: the field is 0 at z = 0**

Note also for z >> R the field becomes $E_{//} = {kq}/{z^2}$ **➔ point charge result.**

Example:

Electric field due to a **segment of a charged ring** with charge **-Q**.



At the on axis point **P**, the vertical components of the field cancel and:

$$dE_{//} = \frac{kdQ}{r^2}Cos(\theta) = \frac{k\lambda ds}{r^2}Cos(\theta) = \frac{k\lambda r d\theta}{r^2}Cos(\theta)$$

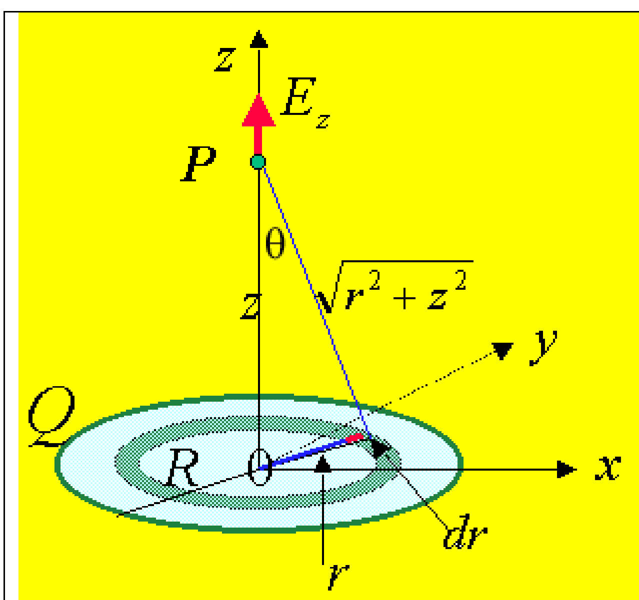
$$E_{//} = \frac{k\lambda}{r}\int_{-\pi/3}^{\pi/3} Cos(\theta)d\theta = \frac{k\lambda}{r}\sqrt{3}$$

Directed toward the segment when Q is negative.

$$\lambda = \frac{-Q}{2\pi r/3} = -\frac{3Q}{2\pi r}$$ **Gives the field in terms of Q and r.**

Example:

Electric field at an **<u>on-axis point due to a charged disk</u>** of radius **R**



The disk of charge corresponds to a summation over rings of charge of width dr.

The surface charge density σ is the charge per area on the disk which if uniform gives dQ = σdA

In terms of r then dQ = σ*2πrdr

As with the ring of charge, the horizontal components of the electric field cancel by symmetry.

$$dE_{//} = dECos(\theta) = \frac{kdQ}{r^2+z^2}Cos(\theta) = \frac{kdQ}{r^2+z^2}\frac{z}{(r^2+z^2)^{1/2}}$$

$$dE_{//} = \frac{z * k\sigma 2\pi r dr}{(r^2+z^2)^{3/2}}$$

$$E_{//} = k\sigma z 2\pi \int_0^R \frac{rdr}{(r^2+z^2)^{3/2}} = k\sigma z\pi \frac{-2}{(r^2+z^2)^{1/2}} \Big|_0^R$$

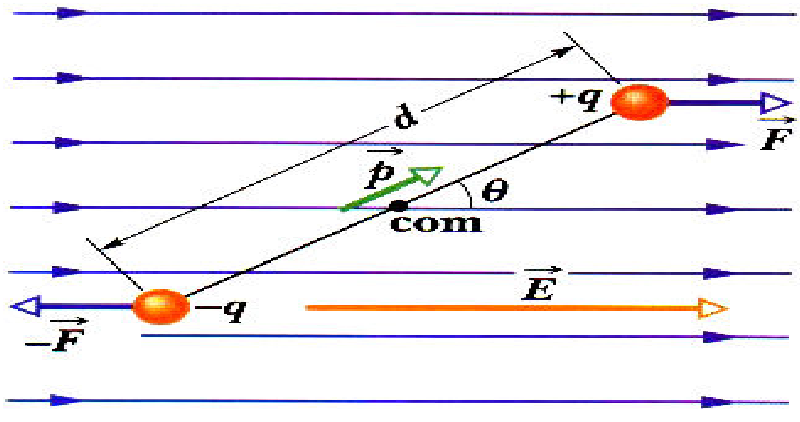
$$E_{//} = -k\sigma z 2\pi \left\{ \frac{1}{\sqrt{R^2+z^2}} - \frac{1}{z} \right\} = k\sigma 2\pi \left\{ 1 - \frac{z}{\sqrt{R^2+z^2}} \right\}$$

The limit **R → ∞** gives the field of an infinite non-conducting sheet of charge: $\boldsymbol{\sigma/2\varepsilon_0}$

Polar Molecules and Torque on a Dipole in an External Field

Water is a **polar molecule** since electrons within the molecule are drawn toward the larger Oxygen atom and a **net electric dipole moment** is established in the molecule.

In an external electric field, an electric dipole moment will experience a net torque:



(a)

(b)

$$\sum_{COM} \tau = -xF_{+}Sin(\theta) - (d-x)F_{-}(Sin(\theta)) = -qEdSin(\theta)$$

Where the last step follows since $\left|F_{+}\right| = \left|F_{-}\right| = qE$

$$\sum_{COM} \tau = -PESin(\theta) = \vec{P} \times \vec{E}$$

The external field aligns the dipole into a position of minimal potential energy:

Potential Energy

The electric potential energy associated with a dipole rotated at a non-zero angle θ in a uniform external electric field is:

$$U = -Work\_done\_by\_the\_field = -\int \tau d\theta = -PECos(\theta)$$

The potential energy is therefore:

$$U = -\vec{P} \bullet \vec{E}$$

This is minimal for $\theta = 0^0$ and maximum at $\boldsymbol{\theta = 180^0}$

Rotating the dipole in an external field ➔ work done by the field:

$$W = -\Delta U = -(U_f - U_i) = -PE\{Cos(\theta_i) - Cos(\theta_f)\}$$