Sound

Sound waves are **longitudinal mechanical waves**. Oscillations of mass elements within the propagating medium are in a direction parallel to the wave propagation direction.

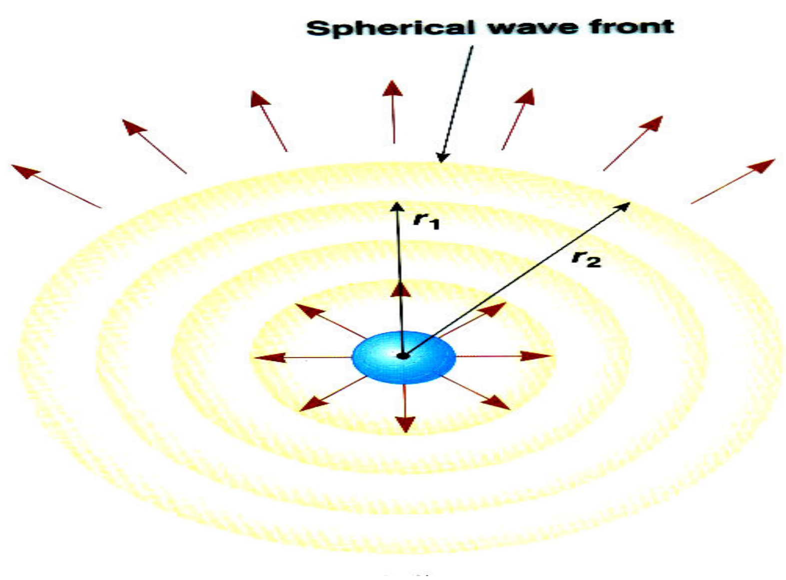
In air, pressure fluctuations either above or below the normal atmospheric pressure correspond to **compressions** and **rarefactions** respectively.

Variations in pressure set molecules of the medium into vibratory motion. In air, these oscillations are eventually detected at the tympanum of the ear as sound.

A source of sound is whatever is producing the compressions / rarefactions within the propagation medium. We typically refer to sound as the vibrations within the **air** that surrounds us, but **sound** is a more general term and is used to refer to longitudinal waves propagating in any state of matter.

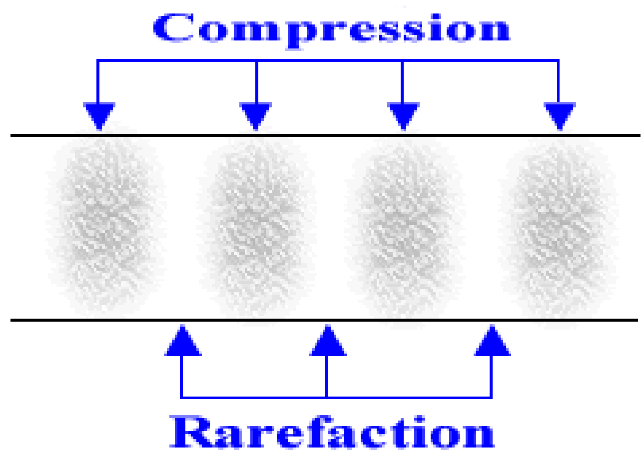
In **3-dimensions** a source emitting **spherical wavefronts** produces sound waves propagating spherically outward from the source at the speed of sound within the medium.

At distances far removed from the source, the spherical nature of the wave is diminished and a **plane wave** approximation is useful.

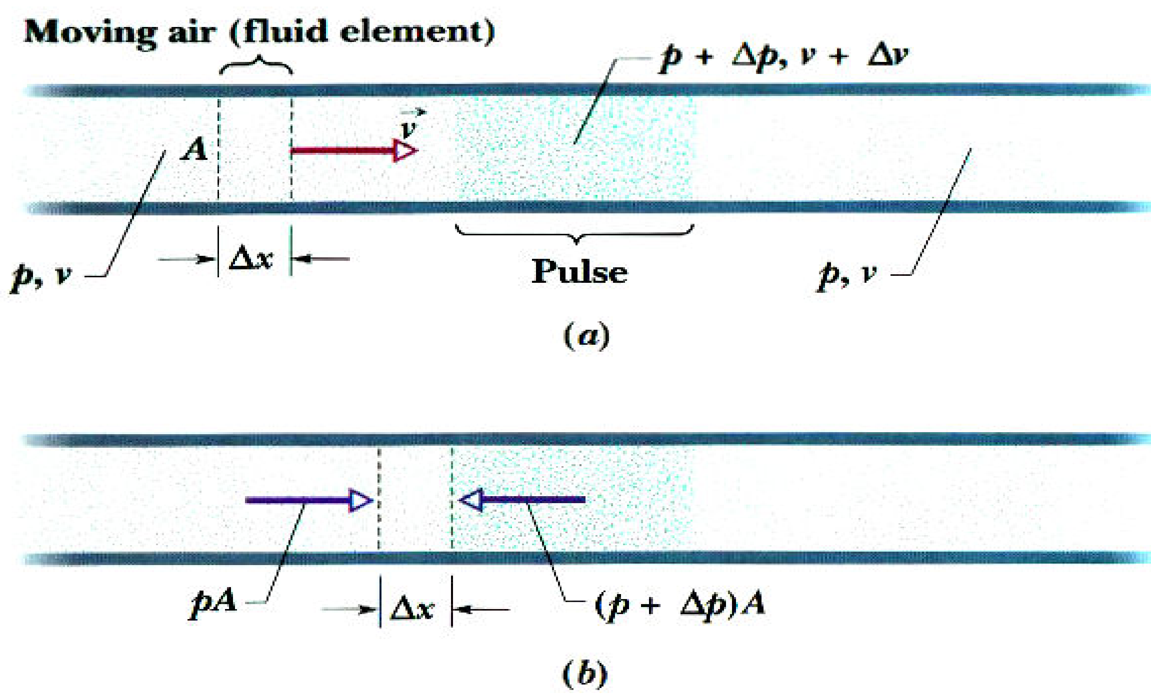


Speed of Propagation

A sound wave propagating in air produces regions of **overpressure** where molecular densities are highest and pressures are above equilibrium pressure.



As sound waves propagate outward, an overpressure **pulse** encounters undisturbed elements of the medium. From the pulse **rest frame**, the overpressure pulse is at rest and the undisturbed air mass element propagates towards this pulse. This is shown below.



Air mass element at pressure p and speed v (speed of sound) collides with the pulse.

The air element mass is $\Delta m = \rho A \Delta x$.

As the front of the mass element encounters the overpressure, Newton's 2nd Law ➔

$$\sum F_x = pA - (p + \Delta p)A = -(\Delta p)A$$

$$-(\Delta p)A = ma = \rho A \Delta x \frac{\Delta v}{\Delta t}$$

$$-(\Delta p)A = \rho A v \Delta t \frac{\Delta v}{\Delta t}$$

$$-(\Delta p) = \rho v \Delta v \quad \Rightarrow \quad -\frac{\Delta p}{\Delta v / v} = \rho v^2$$

Air outside the pulse has volume $V = A v \Delta t$ and is compressed an amount $\Delta V = A \Delta v \Delta t$ inside the overpressure. The denominator above is then:

$$\Delta v / v = \Delta V / V$$

$$-\frac{\Delta p}{\Delta V / V} = \rho v^2$$

From elasticity, $-(\Delta p) = B\dfrac{\Delta V}{V}$ where B is the medium bulk modulus

$$B = \rho v^2$$

$$v = \sqrt{B/\rho}$$

Velocity is dependent on the **medium elasticity** and its **inertial properties**.

Higher **B** ➔more incompressible medium ➔greater **v.**

Increased $\mathbf{V_{sound}}$ is also observed in low-density gases such as Helium. In these gases, increasing v ➔ increasing frequency f since $v = f\lambda$.

In **liquids and solids,** $\mathbf{V_{sound}}$ is relatively high in comparison to its speed in a gas since these states of matter are less compressible.

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For **ideal gases** undergoing an adiabatic compression [no heat exchange],

$$pV^{\gamma} = const.$$ Where $\gamma = \dfrac{C_p}{C_V}$

$$\Delta p V^{\gamma} + p\gamma V^{\gamma-1}\Delta V = 0$$

$$B = -(\Delta p)\frac{V}{\Delta V} = \frac{p\gamma V^{\gamma}}{V^{\gamma}} = \gamma p$$

Bulk modulus = f (pressure)

$$v = \sqrt{B/\rho} = \sqrt{\gamma p/\rho} = \sqrt{\frac{\gamma RT}{M}}$$

Here M is the gas molecular weight.

For sound waves within a **solid rod**, the Young's Modulus is the relevant elastic term:

$$v = \sqrt{Y/\rho}$$

Other Sound Data

In air at 273 K, the speed of sound is **331 m/s**.

A temperature dependence: **V = (331 + 0.6*T) m/s** where **T** is in Celsius results in a speed of sound at room temperature 293 K of 343 m/s.

The ear audible range is for **20 to 20,000 Hz**.

Aging reduces the high frequency audible limit to about 10,000 Hz

Above 20,000 Hz is termed **ultrasonic**. Sound waves at these frequencies have medical imaging, ranging and sonar applications for example.

Below 20 Hz, the **infrasonic** sound waves are used in seismic activity detection and are frequently generated by heavy machinery.

Instrument **timbre** derives from a combination of integer multiples of the instrument fundamental frequency. **White noise** is an equal mix of all audible frequencies.

Traveling Sound Waves

As sound waves propagate, the displacement of a medium molecule as it encounters an incident sound wave is related to the pressure variation in the medium.

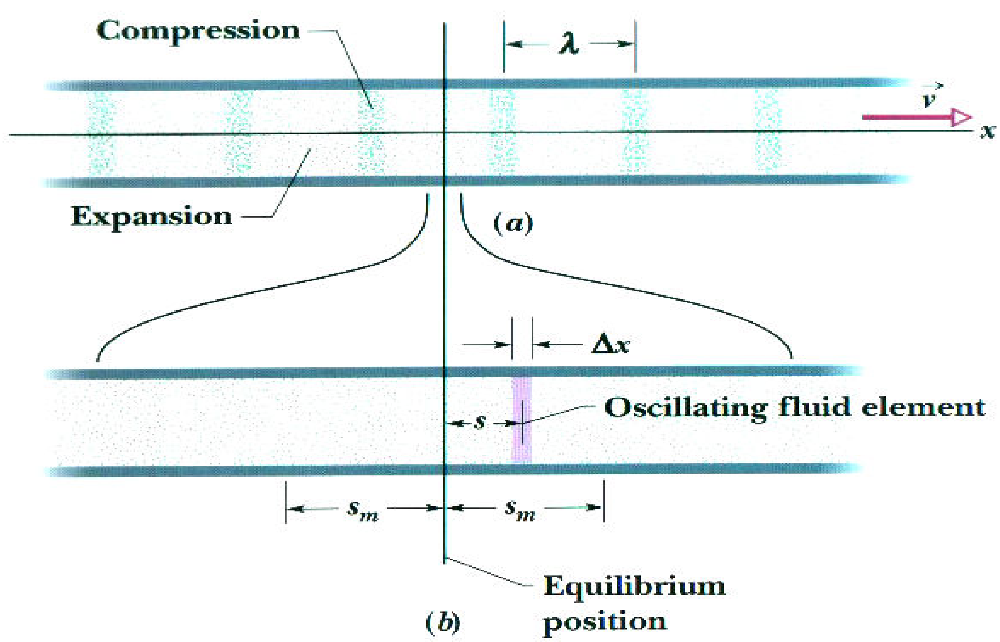
If the sound wave is generated from a source executing simple harmonic motion, then the medium mass element longitudinal displacement is also simple harmonic:

$$s(x) = s_m Cos(kx - \omega t)$$

$$s_m = Amplitude \qquad k = {}^{2\pi}\!/_{\lambda}$$

$$\lambda = Wavelength\_of\_Sound$$

$$\omega = 2\pi f$$



In terms of pressure fluctuations, $$-(\Delta p) = B\frac{\Delta V}{V}$$

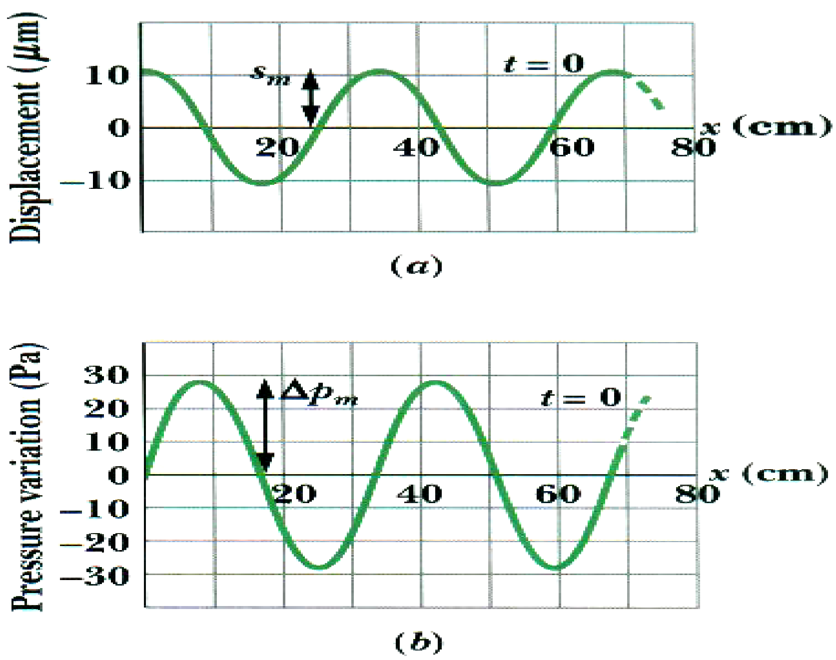
$$V = A\Delta x \qquad \Delta V = A\Delta s$$

$$\Delta p = B\frac{\Delta s}{\Delta x} \Rightarrow dp = B\frac{ds}{dx} = B\{ks_m Sin(kx - \omega t)\}$$

$$\Delta p = \Delta p_m Sin(kx - \omega t)$$

$\Delta p_m = Bks_m = v^2 \rho ks_m$ ➔ Maximum pressure variation.

Note here the 90^0 phase shift between Δp and $s(x)$



Displacements at minimum imply **pressure variations** are maximal.

Intensity

The **intensity** of a sound wave is the time rate of energy transfer from the wave crossing over a unit surface area perpendicular to that surface area at the point of detection.

For a point source emitting spherical wave fronts:

$$I = \frac{P_{Source}}{4\pi r^2}$$

Here $\mathbf{r}$ is the radial distance from the source and P_{Source} is the source power rating.

Units of intensity are $[I] = \frac{Watts}{meter^2}$ and for a point source, **intensity falls off** as $1/r^2$.

The energy of the wave and its intensity are proportional to the square of the wave amplitude [see below] such that the **amplitude falls off** as $1/r$.

The **human ear** has a sound intensity sensitivity range at 2k-4k Hz of

$$10^{-12}\frac{W}{m^2} < I < 1\frac{W}{m^2}$$

1 W/m² corresponds to the threshold of pain.

The **human ear** has a pressure variation sensitivity range at 1kHz of about

$$10^{-5}\,Pa < \Delta p_m < 28\,Pa$$

28 Pa corresponds to the threshold of pain.

Energy and Power

The kinetic energy transferred to the medium is:

$$dk = \frac{1}{2} dm v_s^{\,2} = \frac{1}{2}(\rho A dx)\{-\omega s_m Sin(kx - \omega t)\}^2$$

dm is the differential mass element and $\mathbf{V_s}$ is the speed of the oscillating element.

$$\frac{dk}{dt} = \frac{1}{2}(\rho A v)\omega^2 s_m^{\,2} Sin^2(kx - \omega t)$$

Averaging this quantity, we need to average the Sine squared function:

$$\overline{Sin^2(kx - \omega t)} = \frac{1}{T}\int_0^T Sin^2(kx - \omega t)dt$$

Let $u = kx - \omega t \rightarrow du = -\omega dt$

$$\frac{1}{T}\int_0^T Sin^2(kx - \omega t)dt = \frac{-1}{\omega T}\int_{kx}^{kx-\omega T} Sin^2(u)du = \frac{-1}{\omega T}\int_{kx}^{kx-\omega T} \frac{1 - Cos 2u}{2} du$$

$$\overline{Sin^2(kx - \omega t)} = \frac{-1}{\omega T}\left(\frac{u}{2} - \frac{Sin(2u)}{4}\right)_{kx}^{kx-\omega T} = \frac{1}{2}$$

$$\left.\frac{dk}{dt}\right|_{AVG} = \frac{1}{4}(\rho A v)\omega^2 {s_m}^2$$

For an oscillator,

$$\left.\frac{dPE}{dt}\right|_{AVG} = \left.\frac{dk}{dt}\right|_{AVG} = \frac{1}{4}(\rho A v)\omega^2 {s_m}^2$$

$$P_{Total} = \frac{1}{2}(\rho A v)\omega^2 {s_m}^2$$

$$I = \frac{P_{Total}}{A} = \frac{1}{2}(\rho A v)\omega^2 {s_m}^2 / A$$

$$I = \frac{1}{2}\rho v \omega^2 {s_m}^2$$

Note again that since $I \propto 1/r^2$ this implies that $s_m \propto 1/r$

Sound Level and the Decibel Scale

By definition (**Alexander Graham Bell**), the **sound level** is related to wave intensity as:

$$Sound\_Level = \beta = 10dB * Log\frac{I}{I_0} \qquad I_0 = 10^{-12}\, W/m^2$$

This definition gives $\beta=0$ for the minimum sensitivity of the **human ear** I_0.

For $I = 1\ W/m^2$, $\beta=120$ **db**, which is the threshold of pain for the human ear.

An increase of 10 dB is roughly equivalent to doubling the sound loudness.

Sound level **differences** are related to wave intensity as

$$\beta_2 - \beta_1 = 10dB * Log \frac{I_2}{I_1}$$

A **dBA** scale weights more heavily the midrange frequencies of audible sound where the human ear is more sensitive.

Musical Sounds

Standing wave resonant sound waves in an organ pipe or a wind instrument cavity are the sustained sounds we associate with these particular instruments.

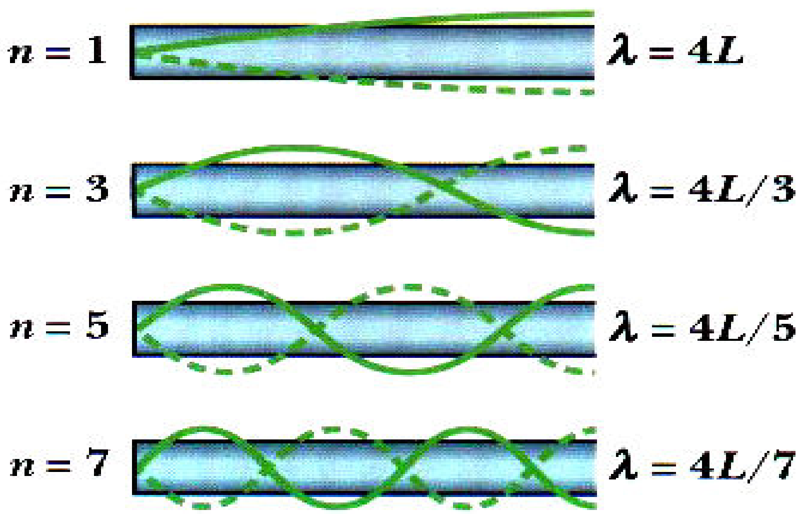
The conditions for which a resonant wave is established in a sound tube of length **L** depends on the boundary conditions at the tube ends:

For a sound tube **closed at one end**, a sound wave node is required at that end and a resonance is setup only if an **odd integral number of $\lambda/4$** wavelengths are in the tube.

If the tube is **open at both ends**, then antinodes are required at both ends since sound may be heard at either end. In terms of pressure variation Δp at the open tube ends, there is none, it's an **ATM**. Minimal Δp ➔ maximal s_m ➔ displacement antinode. Resonant conditions within an open ended tube requires an **integral number of $\lambda/2$**.

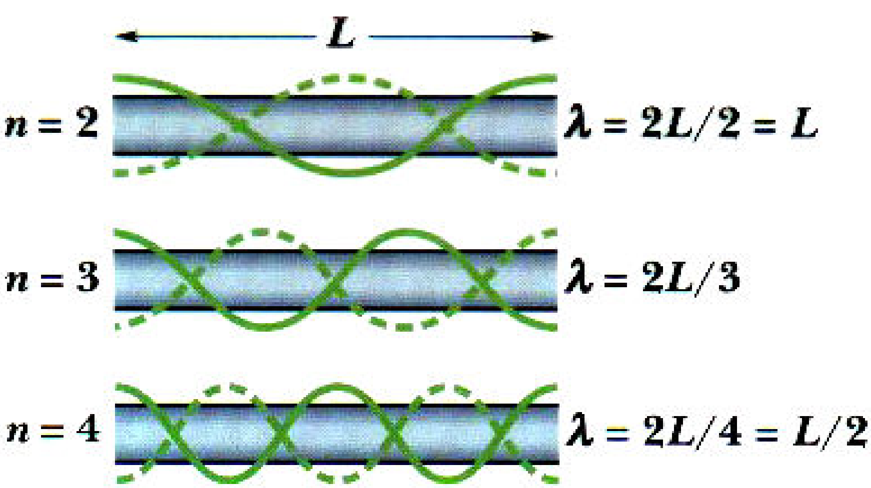
Tube Closed At One End

$$L = n\frac{\lambda}{4} \qquad \mathbf{n = 1,3,5\ldots} \qquad f = \frac{v}{\lambda} = n\frac{v}{4L}$$



Tube open at both ends

$$L = n\frac{\lambda}{2} \qquad \mathbf{n = 1,2,3\ldots} \qquad f = \frac{v}{\lambda} = n\frac{v}{2L}$$



Sound Quality and Harmonic Number

n is the **harmonic number**: **n=1** is the **fundamental** **n>1** are **overtones**.

The frequency range available to an instrument is principally determined by **L**.

The fundamental and first few harmonics add to create the **timbre** of an instrument.

Overtone amplitudes vary from one instrument to the next giving each instrument a unique sound quality. The timbre of an **oboe** A# is easily distinguished from a **flute** A#.

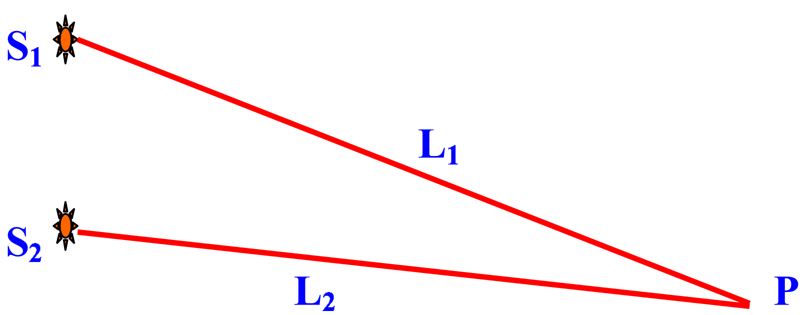
For **stringed instruments**, an integral number of $\lambda/2$ within the fixed ends will produce a resonant wave which may be amplified by a sounding box as with a guitar for example.

$$L = n\frac{\lambda}{2} \qquad \mathbf{n = 1,2,3\ldots} \qquad f = \frac{v}{\lambda} = n\frac{v}{2L}$$

Sound Wave Interference

Sound wave interference occurs when two or more waves simultaneously propagate in the same region of space.

For two in phase sound sources $\mathbf{S_1}$ and $\mathbf{S_2}$ emitting with the same frequency, interference at any point **P** away from the two sources results from a **path length difference** from the sources to that point.



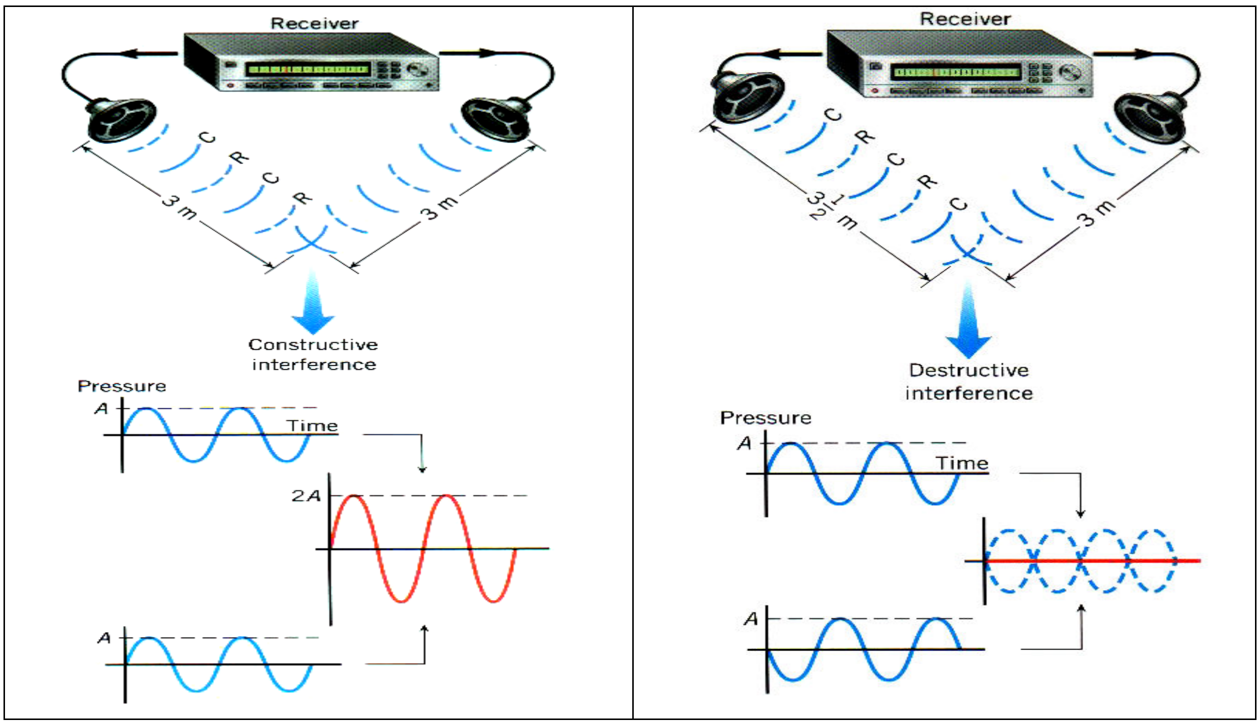
Interference at point **P** depends on the difference $|\mathbf{L_1} - \mathbf{L_2}|$.

If $\mathbf{L_1 = L_2}$, then the two waves produce **<u>fully constructive interference</u>** at point **P.**

For $|\mathbf{L_1 - L_2}| = \lambda/2$, the waves arriving at point **P** are $\boldsymbol{\pi}$ out of phase with each other and **<u>fully destructive interference</u>** is produced at point **P**.

<u>Partially destructive interference</u> will occur for $\mathbf{0} < |\mathbf{L_1 - L_2}| < \lambda/2$

Consider an interference from the **1m** wavelength sources below:



$|\mathbf{L_1 - L_2}| = \mathbf{n} * \lambda$ $\quad$ $\mathbf{n = 0,1,2,\ldots}$ ➔ Fully constructive interference.

$|\mathbf{L_1 - L_2}| = (\mathbf{n+0.5}) * \lambda$ $\quad$ $\mathbf{n = 0,1,2,\ldots}$ ➔ Fully destructive interference.

The **<u>phase difference</u>** as the two waves arrive at a distant point **P** is:

$$\frac{\Delta L}{\lambda} = \frac{\Delta \varphi}{2\pi} \Rightarrow \Delta \varphi = 2\pi \frac{\Delta L}{\lambda}$$

Beats

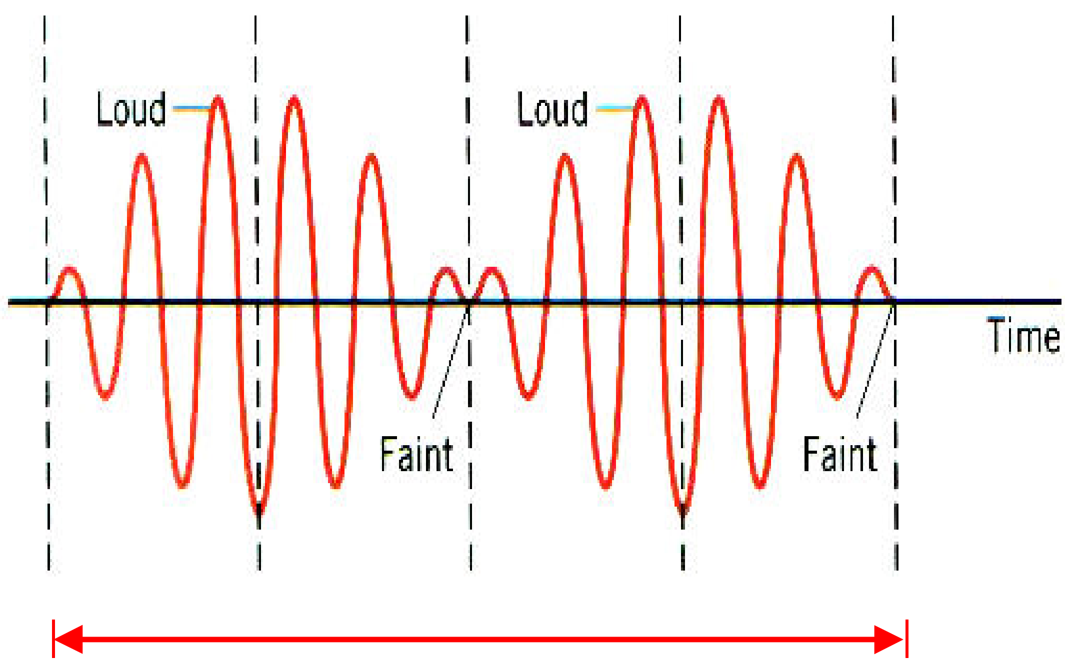
For two interfering waves of the same amplitude, and of **slightly different frequency**, a superposition yields a **beat wave** with a frequency that is an average of the two separate frequencies and an amplitude oscillating at frequency **$|f_2 - f_1|$**, called the **beat frequency**.

$$s(x=0) = s_m\{Cos(\omega_1 t) + Cos(\omega_2 t)\}$$

$$s(x=0) = 2s_m Cos[\frac{1}{2}(\omega_1 - \omega_2)t]Cos[\frac{1}{2}(\omega_1 + \omega_2)t]$$

$$s(x=0) = 2s_m Cos(\omega' t)Cos\frac{1}{2}(\omega_1 + \omega_2)t$$

During the time the envelope cycles once the amplitude has peaked twice. The 'beat' angular frequency is then **$(2*\omega')$** and equal to **$2\pi*|f_2 - f_1|$**



Beat frequency is the reciprocal of one-half this total time interval.

Doppler Effect

Motion of an observer with respect to a fixed sound source or motion of the sound source with respect to a fixed observer or any combination of the two motions will result in a frequency shift in the detected sound wave that depends on the relative velocity of the observer and/or source with respect to the **stationary air**:

This frequency shift is known as the **Doppler Effect** and may be quantified only for **speeds less than the speed of sound V**:

The detected frequency f' in terms of the original source frequency f

$$f' = f\left[\frac{V \pm V_D}{V \pm V_S}\right]$$

$$V = Speed\_of\_Sound$$

$$V_D = Detector(Observer)\_Speed$$

$$V_S = Speed\_of\_Source$$

Relative motion of the observer towards the source will shift the sound wave to higher frequencies :

$$f' > f \Rightarrow Use\_either\_+V_D\_or\_-V_S$$

Relative motion of the observer away from the source will shift the sound wave to higher frequencies :

$$f' < f \Rightarrow Use\_either\_-V_D\_or\_+V_S$$

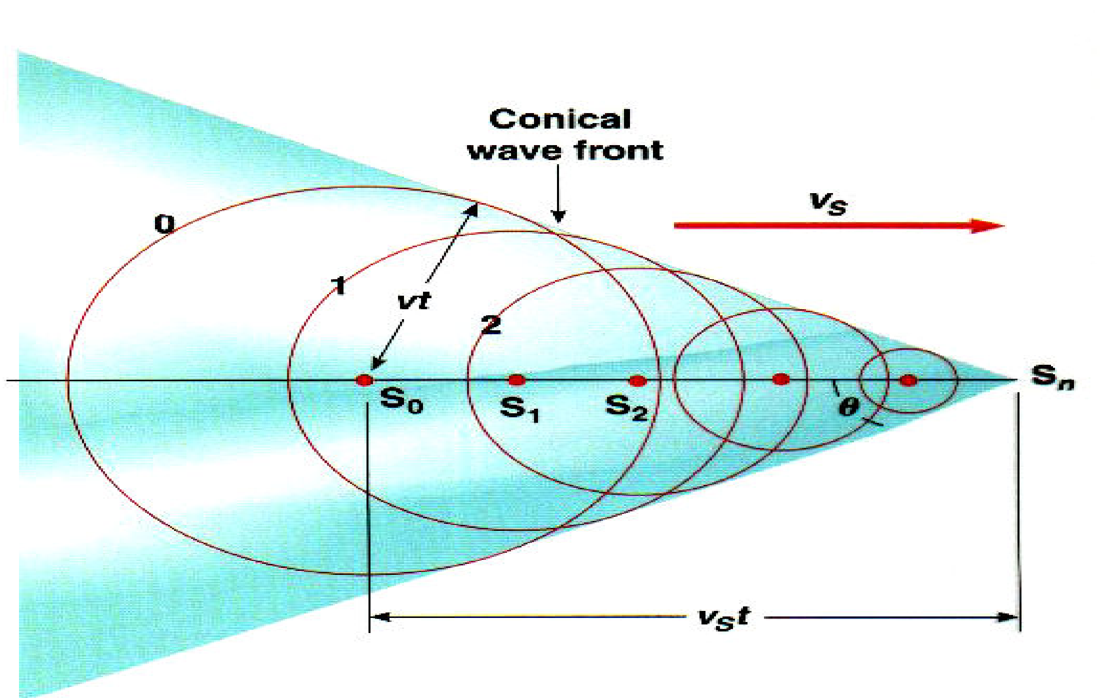
Shock Waves

Objects having speed **greater than or equal to** the speed of sound **V** create sound wavefronts that are concentrated along the surface of a **'Mach Cone'**.

Concentration of sound wave energy along the wavefronts produces a large pressure fluctuation on the Mach Cone. Observers are jolted with a Sonic Boom shock wave.

$$V_S = Speed\_of\_Source$$

$$V = Speed\_of\_Sound$$



$$Sin(\theta) = \frac{Vt}{V_S t} = \frac{V}{V_S}$$

The ratio V_S / V is called the **Mach number**.