

Wave Optics

As the wavelength of light becomes comparable to the dimensions of the apertures and obstacles in an optics path, effects such as interference and diffraction are observed.

In geometric optics, light rays either pass by an obstacle or are reflected off a surface on the obstacle. The bending of light around obstacle edge (diffraction) is not predicted.

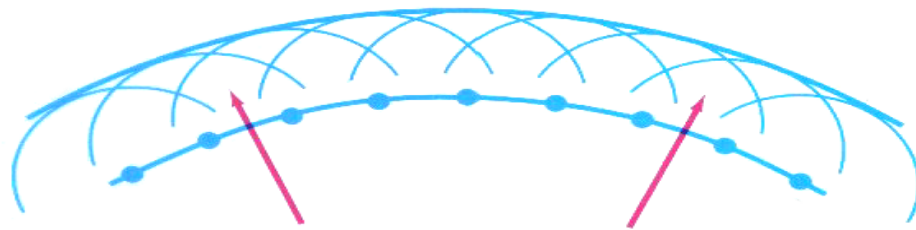
To describe diffraction as observed, a wave theory of light is required that reproduces the geometric optics results, and from which may be extracted analytic results about interference and diffraction phenomena in terms of the light wavelength.

The theory starts with a wavefront that is subsequently partitioned into an infinite number of point sources in a principle put forth by Christian Huygens in 1678.

A wavefront, you may recall, is constructed by connecting the in-time crests of waves emitted at a spherical point source.

A spherical wavefront is then approximated as a plane wave if the observation point is far removed from the source.

Huygens' Principle: all points on the wavefront serve as point sources of spherical secondary wavelets. As the wavefront progresses in time the wavefront position is defined by the surface tangent to the wavelets.



Source S

Reflection and Wave Theory

Both geometric optics and wave optics predicted the Law of Reflection so there was nothing obviously advantageous in the wave theory approach to the topic of reflections.

Upon closer inspection, however, recalling the general analytic form for a traveling wave

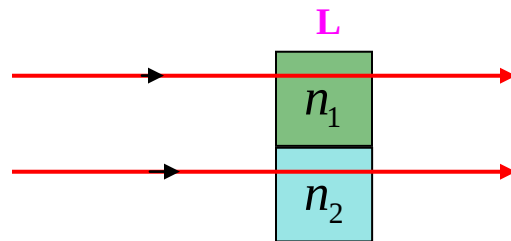
$ASin(kx - \omega t + \phi)$, the reflection of an electromagnetic wave is observed to undergo a phase shift at the reflecting surface under certain circumstances. Then since ray optics did not concern itself with phase angles ϕ , wave theory was more complete.

Electromagnetic waves leaving a medium of refraction index n_1 and entering a medium of **higher** index of refraction n_2 is analogous to a mechanical string wave moving into a region of greater mass density μ , the result is an inverted reflection at the interface. For light, this inversion represents a phase shift π between reflected and incident waves.

If n_1 is **greater** than n_2 , no phase shift takes place.

Consider two waves of the same wavelength and phase passing through different

mediums n_1 , and n_2



The number of wavelengths within L is $\frac{L}{\lambda_n}$ where $\lambda_n = \frac{\lambda}{n}$ in each medium.

The difference resulting is $N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1)$

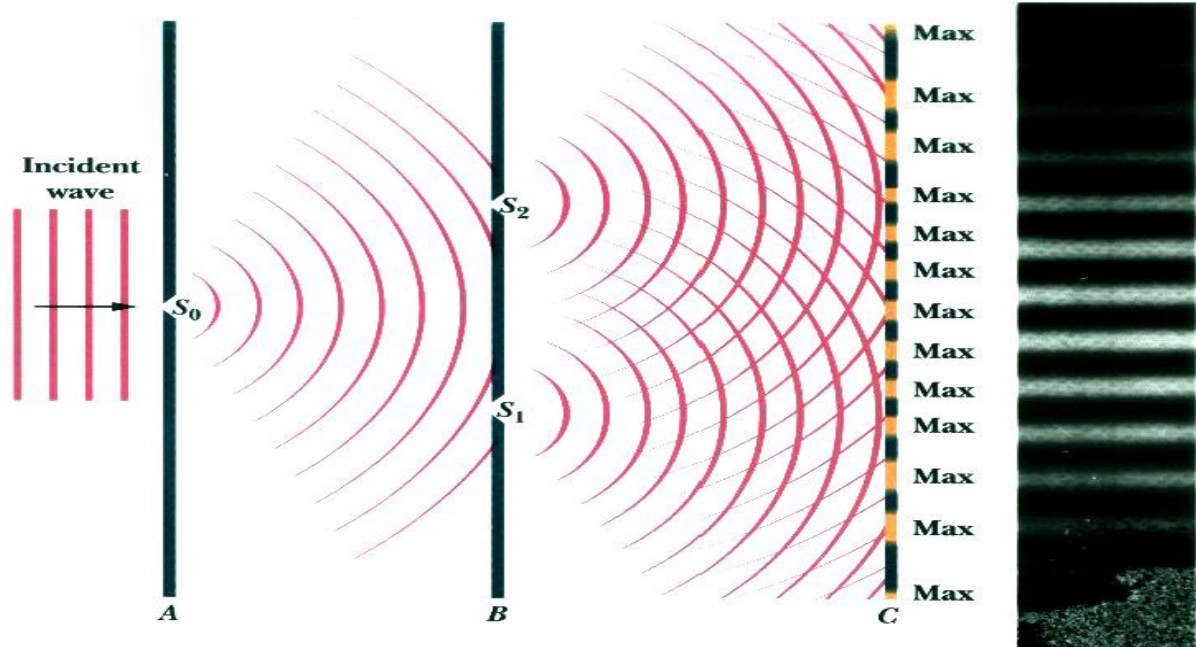
If this is a **whole number** of wavelengths, the two waves exit the medium in phase.

If this is an odd multiple of **half integer** wavelengths, then a π shift has occurred.

Young's Interference - Double Slit Interference

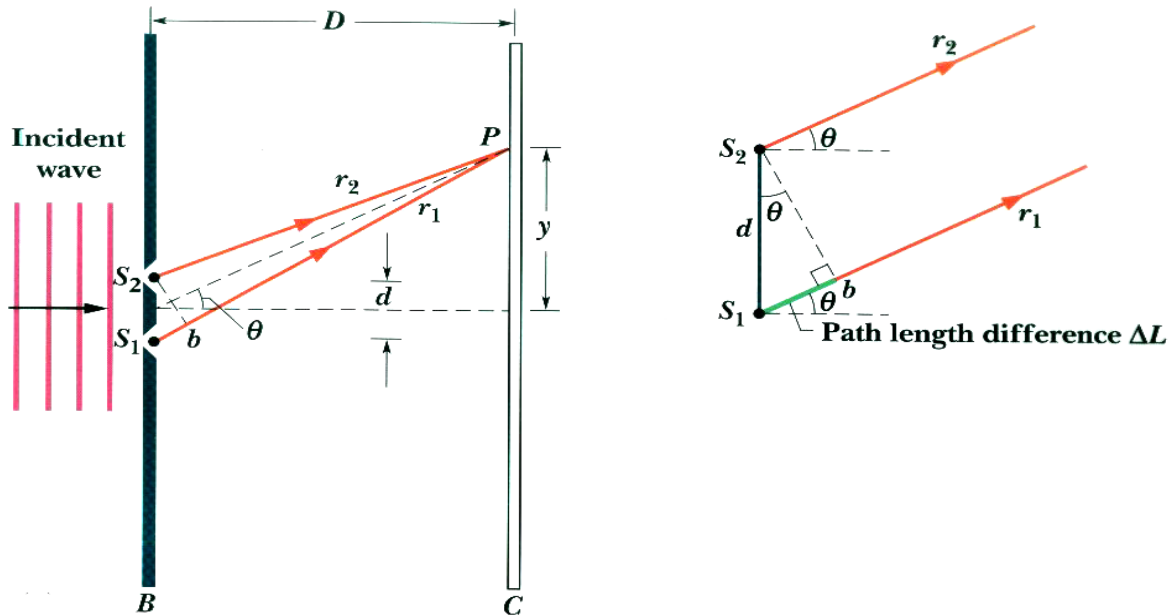
In 1801, Thomas Young showed the wave theory of light was correct by demonstrating that when two or more electromagnetic waves enter into the same region of space at the same time interference takes place.

Monochromatic, coherent incident plane waves will diffract and interfere creating bright band intensity maxima and dark fringe intensity minima at the screen location.



Distinguishing between fully constructive, fully destructive, and partially destructive interference at any point along the screen depends on path length differences to the screen

from the two source points S_1 and S_2 .



Interference effects at point **P** are due to the difference in path length ΔL in relation to the light wavelength.

For the condition $D \gg d$, it can be seen that $\Delta L = d \sin(\theta)$

θ = Angle measured with respect to the central axis

Maxima occur when:

$$\Delta L = d \sin(\theta) = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

Minima occur when:

$$\Delta L = d \sin(\theta) = (m + \frac{1}{2})\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

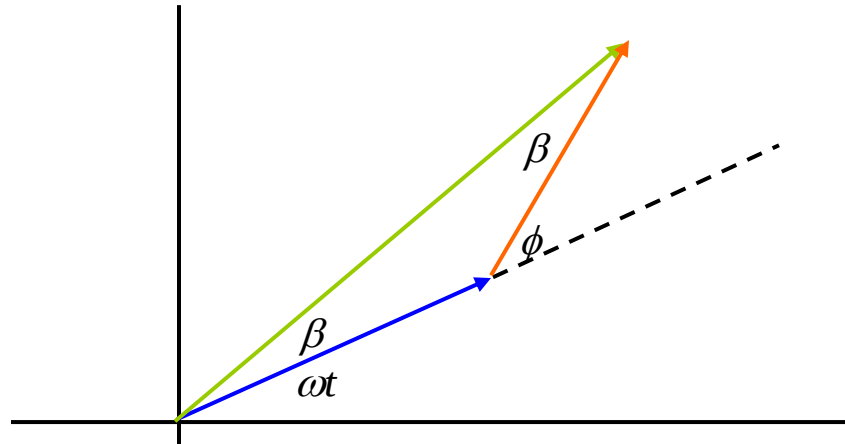
θ Locates a particular maxima or minima and $\theta = 0$ is the central maximum.

m is the order number.

For small angles θ , the vertical distance to a bright band is $y_m = D \frac{m\lambda}{d}$

Screen Intensity $I(\theta)$

A Phasor addition of same-amplitude E_0 , out-of-phase waves arriving at point **P** may be evaluated to give the **interference pattern intensity** as a function of θ



$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin(\omega t + \phi)$$

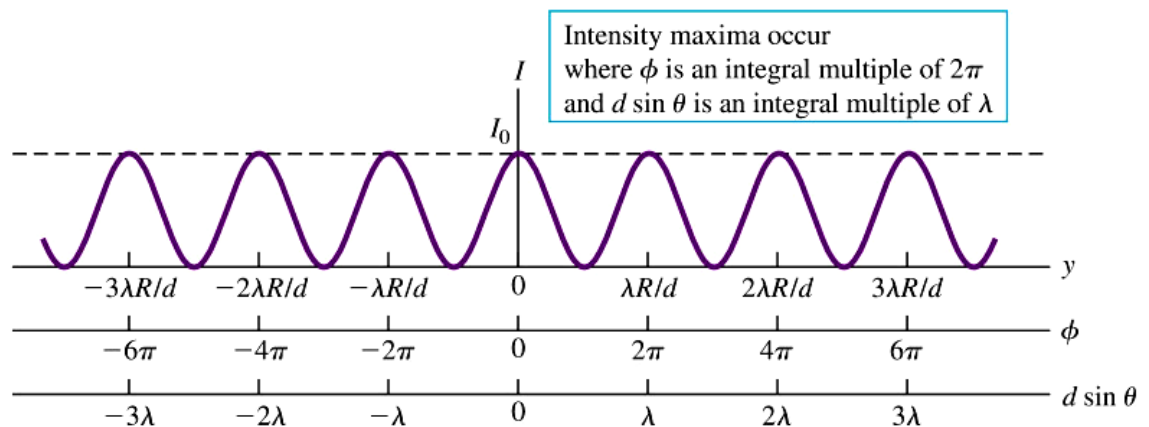
$$E = 2E_0 \cos(\beta) = 2E_0 \cos\left(\frac{\phi}{2}\right)$$

$$E^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)$$

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Relating the phase difference ϕ and the angle θ

$$\phi = \frac{2\pi}{\lambda} \Delta L = \frac{2\pi d \sin(\theta)}{\lambda}$$



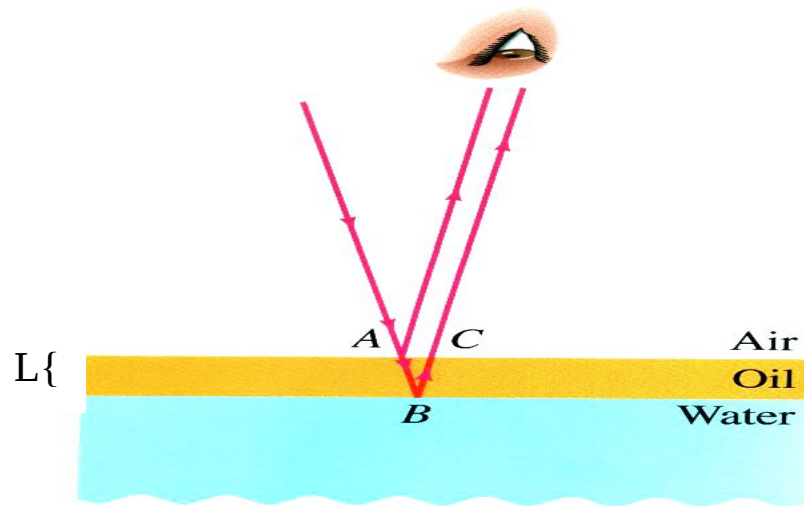
Pattern is lost if sources are incoherent, and bright bands require $m\lambda < d$

Thin Film Interference

Soap bubbles, oil slicks, Morpho-Butterfly wings, etc exhibit thin film interference

Thin → thickness of film is comparable to λ

When incident waves undergo a reflection phase shift from the top layer of a thin film and the reflected waves are subsequently superposed with reflected waves from the back of the thin film, interference is possible.



Superposition gives totally constructive interference if
$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2}$$

The phase difference associated with this path length difference combines with the π phase shift from the front surface to result in a constructive interference wave.

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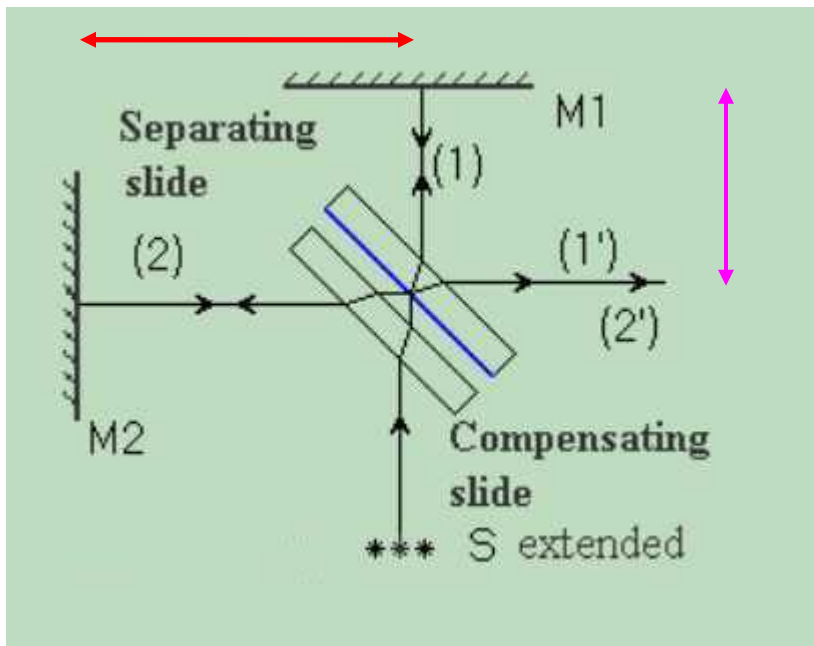
The phase difference associated with this path length difference combines with the π shift from the front surface to produce a destructive interference wave.

If $L < 0.1\lambda$, then $\Delta\phi = \pi$ from the incident wave reflection only and a 'dark film' is produced.

If n_1 is greater than n_2 , no reflection phase shift takes place.

Interferometry

Interferometers exploit light wave interference effects allowing lengths or changes in length to be measured with a high degree of accuracy.



Light from source S encounters a compensator and a beam splitter in the center of the apparatus.

The path length difference [Red -Pink] at the observation point $1', 2'$ determines the type of interference at this point.

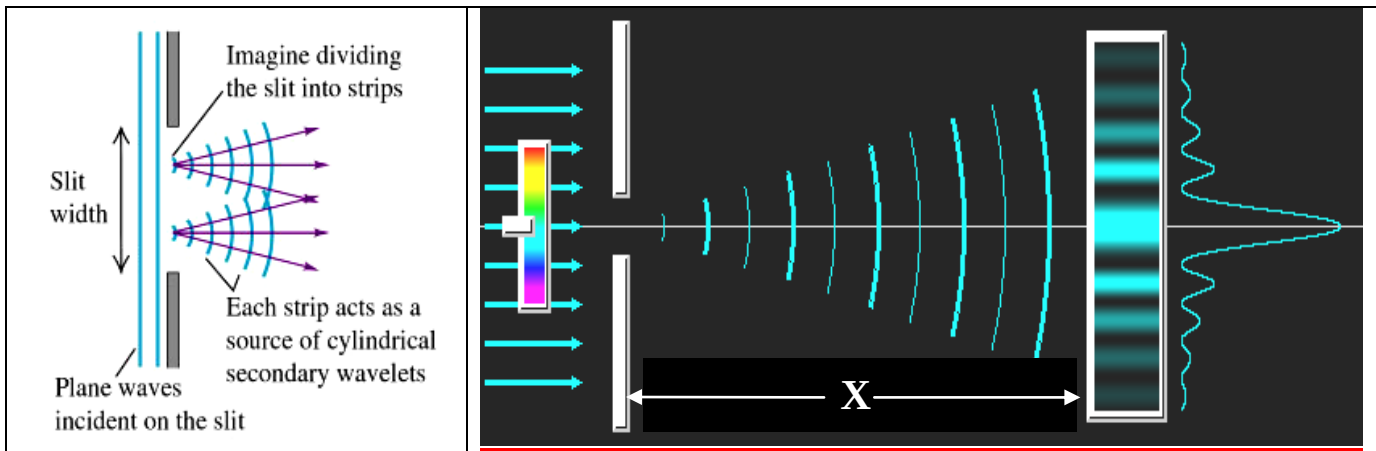
By fixing one of the two mirrors and moving the other mirror $\frac{\lambda}{4}$, a bright interference band at the observation point changes into a dark fringe at that point.

By measuring the distance through which a mirror is moved in order to change the band pattern at the observation point, a measurement of the light wavelength is possible.

If an interferometer is adjusted such that interference is fully constructive at the observation point, then any deviation in either arm length will change the fringe pattern making interferometers extremely precise tool for determining small distance variations.

Single Slit Diffraction

Diffraction or the bending of light at an obstacle edge or corner may be understood from Huygens' principle. If the light source is monochromatic point source light incident on a single slit of width **a**, then a Fraunhofer diffraction pattern is observed on a distant screen.



There is an intensity maximum at $\theta = 0^\circ$ and dark fringe locations are given by:

$$a \sin(\theta) = m\lambda \quad m = \pm 1, \pm 2, \pm 3 \dots$$

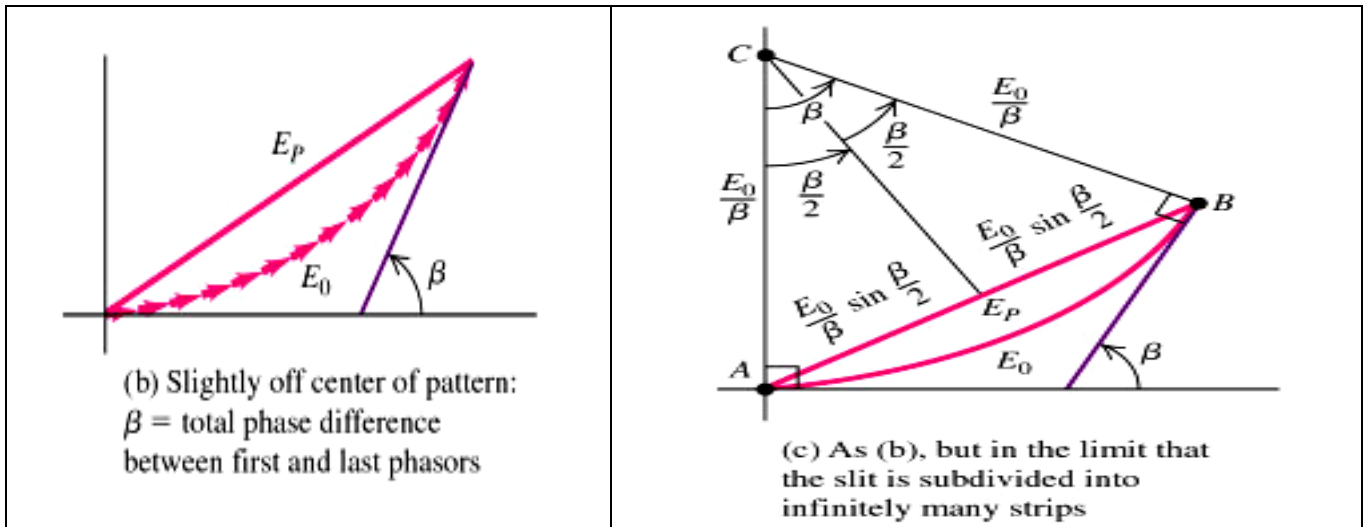
Central maximum width decreases as a/λ is increased.

For small angles, $\theta = m\lambda/a$ where θ is in radians. Note for $a < \lambda$ the effect is lost.

The vertical distance to a dark band from the central axis is $\sim y = x * m\lambda/a$.

Intensity in the Single Slit Pattern

Using wavelet phasors, the superposition at an observation point P is constructed:

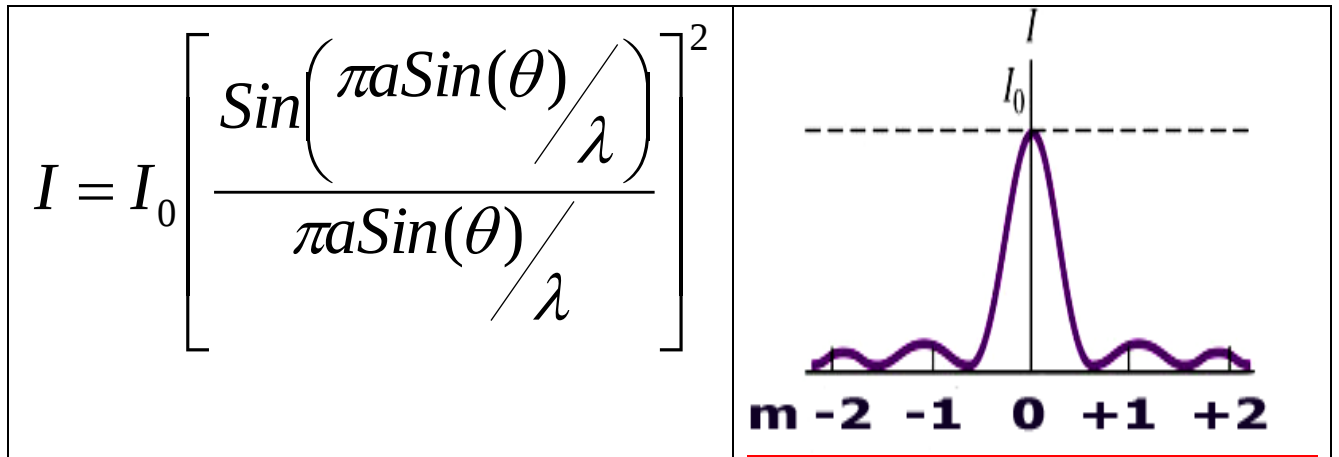


For the quadrangle interior angles summing to 360° , and using $\mathbf{s} = \mathbf{r}\theta$

$$E_p = E_0 \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}$$

$$I = I_0 \left[\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} * \text{path_length_difference} = \frac{2\pi}{\lambda} a \sin(\theta)$$



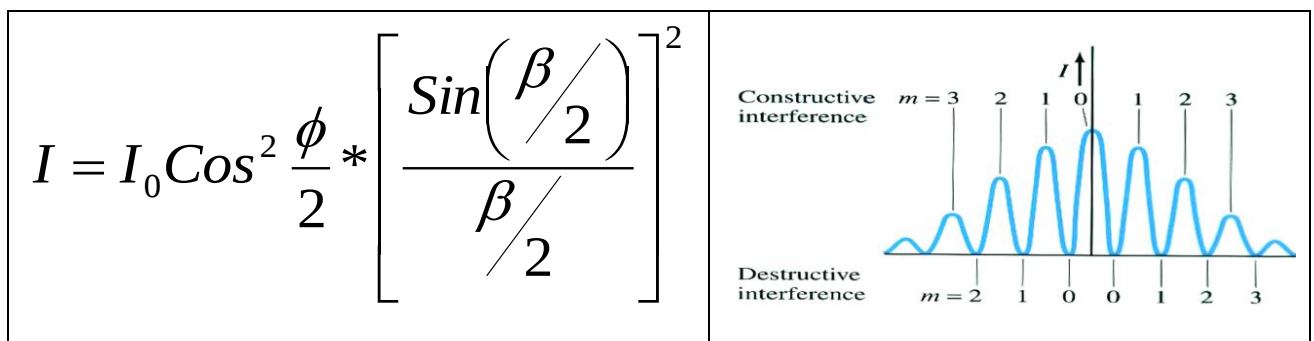
The values of maxima intensity are well approximated as

$$I_m = \frac{I_0}{\left(m + \frac{1}{2}\right)^2 \pi^2}$$

Two-Slit Interference with Finite-width Slits

A more realistic two-slit apparatus has slits of finite dimension and the resulting interference pattern is altered by diffraction effects.

The intensity at the screen is proportional to the product of the double-slit ideal case and the single-slit diffraction intensity. Interference maxima falling into diffraction minima are **missing** and occur for **$d = na$** or **$m_i/m_d = d/a$** .



$$\phi = \frac{2\pi d \sin(\theta)}{\lambda}$$

$$\beta = \frac{2\pi}{\lambda} a \sin(\theta)$$

Diffraction Gratings

A transmission grating consists of a large number of machined slits within a small length of metallic base. The result for finding principal maxima on the screen is:

$$d \sin(\theta) = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

d in this case is the center-center separation distance between slits.

As the number of slits **N** increases, the principal peaks are sharper and more minima **(N-1)** occur between any two principal maxima. Secondary maxima intensity are also diminished as **N** is increased.

Total power at the screen is proportional to **N**, principal maxima height is proportional to **N²**, such that principal maxima width are proportional to **1/N**.

Spectroscopy

White light incident on a diffraction grating will be split into a spectrum of colors at each principal maximum. In absorption spectroscopy, elements absorbing at particular frequencies are indicated by missing lines in the diffraction pattern.

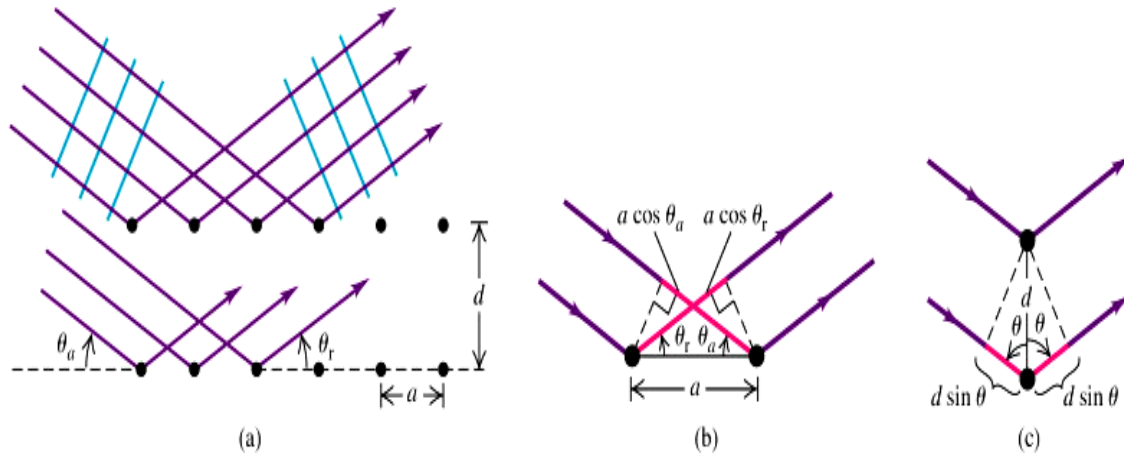
The chromatic resolving power of a spectrograph is a measure of how well the instrument can distinguish between adjacent wavelengths **$\Delta\lambda$** .

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

Higher orders and gratings with a large number of slits have better resolution.

X-Ray Diffraction

The diffraction of x-rays from a crystal lattice array was first performed in 1912 by Von Lau, Friederich and Knipping.



The Bragg condition for constructive interference requires:

$$2d \sin(\theta) = m\lambda \quad m = 0, 1, 2, \dots$$

θ measured with respect to the crystal surface.

Raleigh Criterion and Resolution

The ability of a lens to separate two point objects with small angular separation into two distinct objects is called the lens resolution.

Viewing the problem as diffraction limited, then the criterion for resolution will be that the second object be separated from the first by an angle greater than the angular width of the central diffraction maximum produced by the first object.

Visually the circular diffraction pattern from a single point source appears as:



The angular half width of the central maximum in this pattern is $\theta = 1.22\lambda/D$

Raleigh Criterion

The diffraction limiting angle is the central maximum half width

$$\theta_R = 1.22\lambda/D$$

In order to resolve two objects distinctly

the condition $\theta_{\min} > \theta_R$ must hold.

