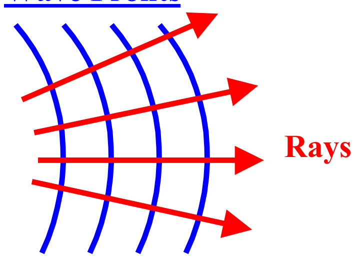
Geometric Optics

If a beam of light is narrow and has short wavelength in comparison to the dimension of any obstacle or aperture in its path, then this beam may be treated as a straight-line **ray** of light and its wave properties for the moment ignored.

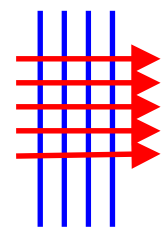
In this approximation, light **rays** are traced through each optics element in the system responding in a mathematically well-prescribed way at each interaction point.

A **wave front** corresponds to the line, sphere or shape formed by connecting the in-time crests in a propagating wave. **Rays** are vectors perpendicular to wave fronts indicating the direction of propagation.

Wave Fronts



Plane Waves



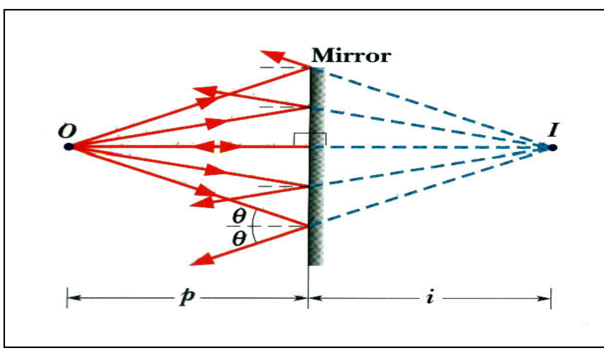
At sufficiently large distances from a source or by considering a small portion of a spherical wave front, propagating waves may be approximated as **plane waves**.

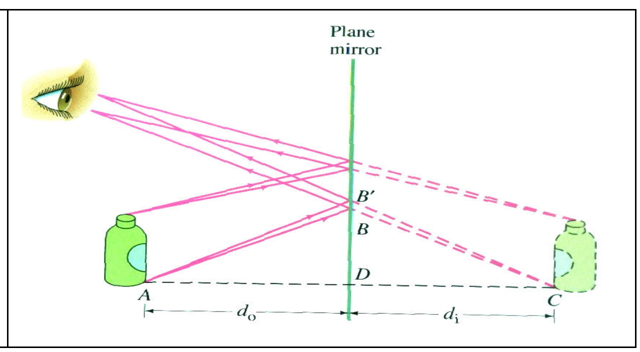
Images

Images are reproductions of objects using light. Either light is reflected from the object to an observer or light is passed through an object for projection.

Virtual Images are optical illusions that exist only when an observer is present. Rays of light do not actually pass through the image location to produce the image.

Images from **Plane Mirrors** appear to originate from a location behind the mirror. Since light does not pass through this image location behind the mirror, the image is virtual.



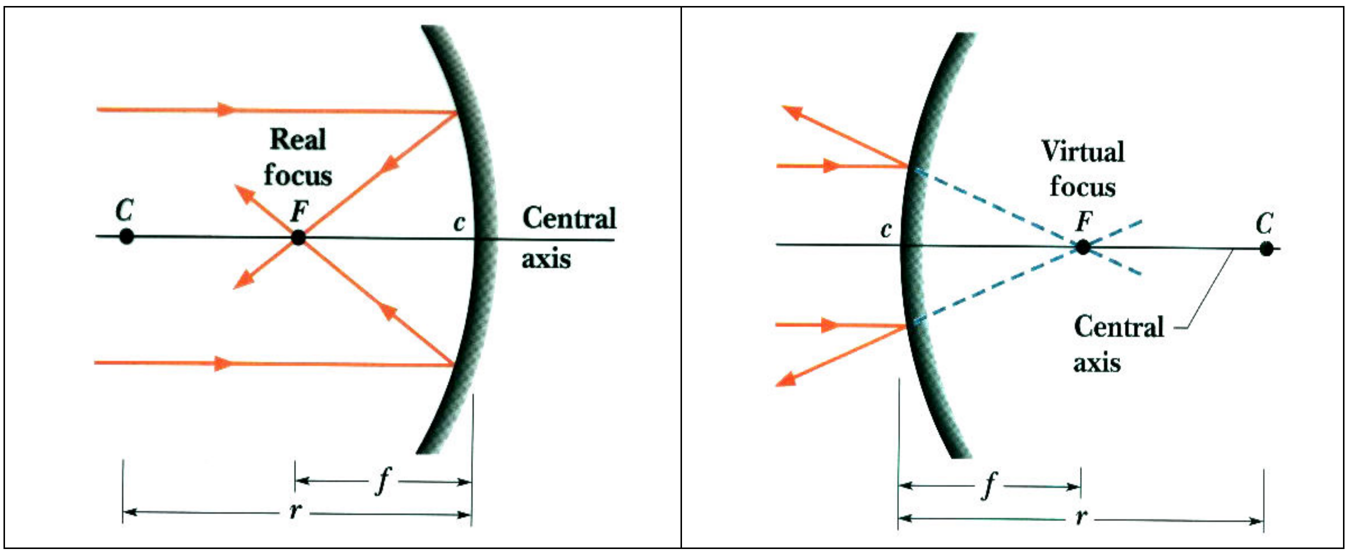


Real Images are formed when light passes through the image location such as the images projected onto a screen, film, etc. An overhead transparency forms a real image.

For Plane Mirrors the image distance i is negative, the object distance p is positive, $|i| = p$ and **lateral magnification** defined $m = -i/p$ is unity.

Spherical Mirrors

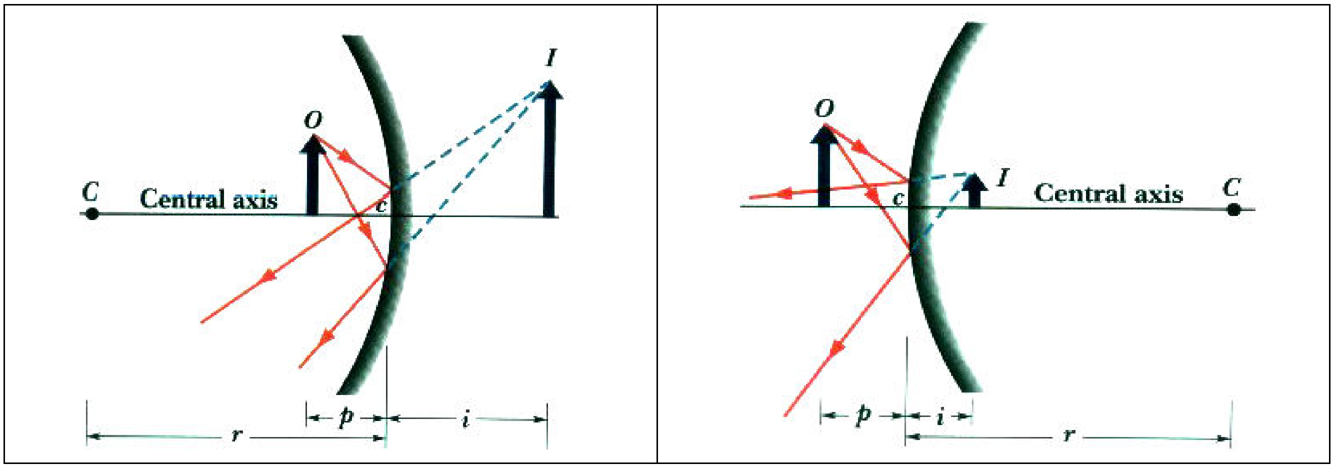
Spherical mirrors are either **concave** or **convex**. The 'realness' of images formed by a **concave** spherical mirror depends on the object position. For **convex** spherical mirrors, images formed are **always** virtual.



F is the **focal point** for each mirror. The **focal length** f is positive (real focal point) for a **concave** mirror and negative (virtual focal point) for a **convex** mirror.

C is the mirror **center of curvature**, and r is positive for the concave mirror and negative for the convex mirror.

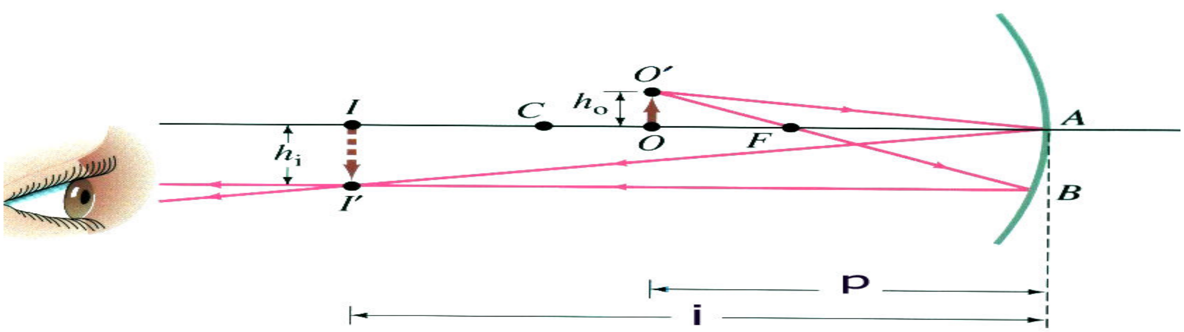
Since light rays coming from an object in front of the mirror have more (less) time to diverge prior to their reflection, the image size produced will be larger (smaller) for a concave (convex) spherical mirror in comparison to images from a plane mirror.



Paraxial rays are approximately focused at the spherical mirror focal point. However, spherical mirrors suffer from **spherical aberration** and not all incident rays focus to a single focal point. **Parabolic** mirrors overcome this problem.

In the paraxial approximation, incident rays are incident at small angles with respect to the **optic axis** and the mirror focal length is related to its curvature: $f = \frac{1}{2}r$

The **mirror equation** relates i, p and f as follows:



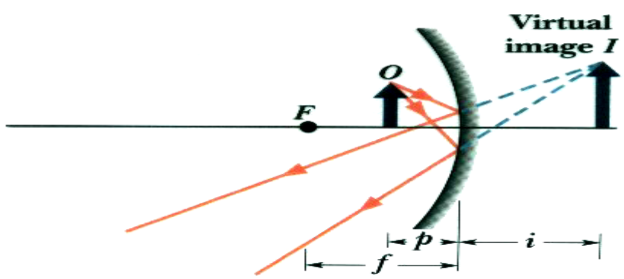
From the similar triangles $O'AO$ and $I'AI$ the result $\frac{h_i}{h_o} = \frac{i}{p}$

From the similar triangles $O'FO$ and FAB the result $\frac{h_o}{h_i} = \frac{p-f}{f}$

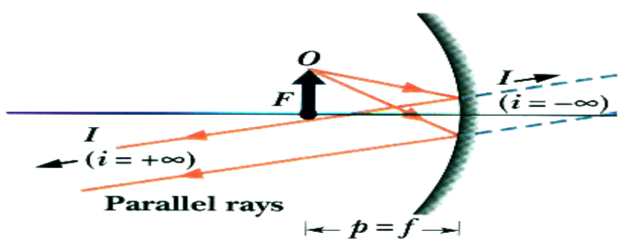
$$\frac{p}{i} = \frac{p-f}{f} \quad \rightarrow \quad \frac{1}{i} = \frac{p-f}{pf}$$

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$$

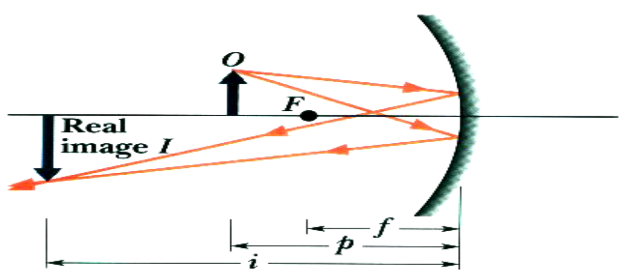
Images formed by a **convex** spherical mirror are always virtual, but images formed by a **concave** spherical mirror depend on the object position relative to F



For objects located **inside** the focal point F, images are upright and virtual. Reflected rays are diverging as they leave the mirror surface and will therefore never converge and/or pass through a real image in front of the mirror.



Locating an object **at** F results in reflected rays that neither diverge nor converge in front of or behind the mirror implying an image is never formed.



If p is outside of F the image becomes real and inverted as shown

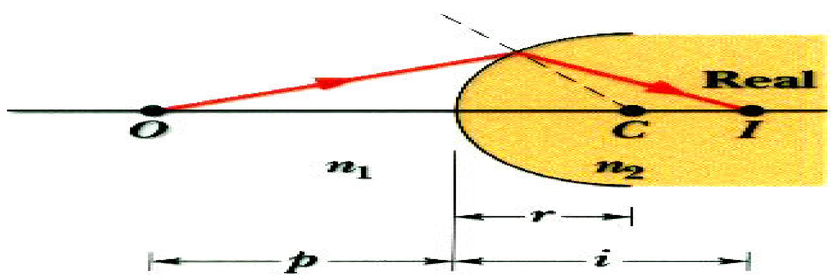
With lateral magnification, $m = -\,{}^{i}\!/_{p}$ a table of mirror results may be compiled:

Mirror Type	Object Location	Image Location	Image Type	Image	Sign of f	Sign of r	Sign of m
Plane	Anywhere	Behind	Virtual	Upright	∞	∞	Positive
Concave	Inside F	Behind	Virtual	Upright	Positive	Positive	Positive
Concave	Outside F	In front	Real	Inverted	Positive	Positive	Negative
Convex	Anywhere	Behind	Virtual	Upright	Negative	Negative	Positive

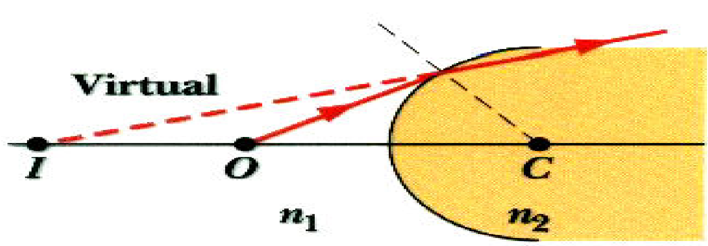
Spherical Refracting Surfaces

Consider rays originating in medium n_1 incident upon and refracted by a single sided spherical lens made of material characterized by an index of refraction $n_2 > n_1$.

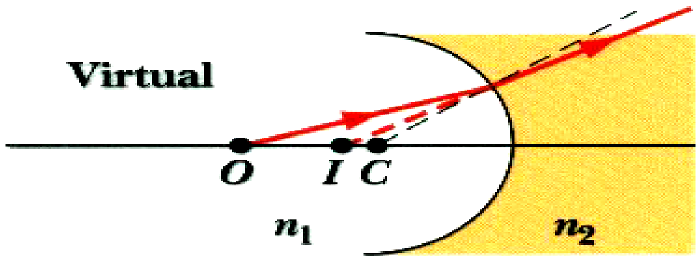
If the spherical lens surface is **convex** and the object is **away** from the front of the lens, then a **real image on the side opposite the object** results.



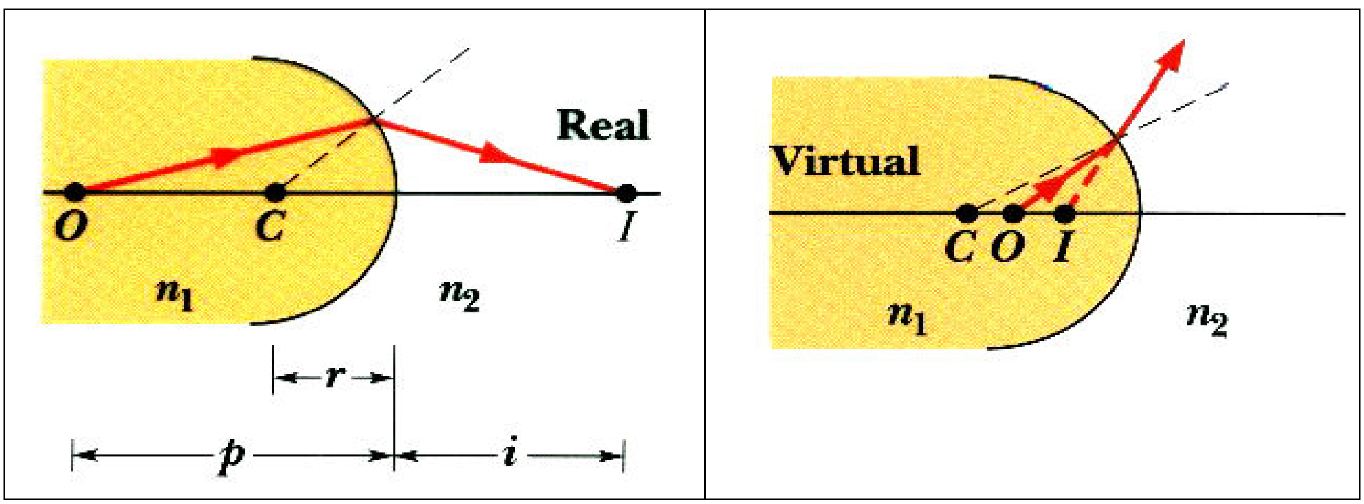
If the object is **close** to the front of the convex lens surface, then refracted incident rays no longer converge to a point within **medium 2** and a **virtual image** is formed on the **same side of the lens as the object**.



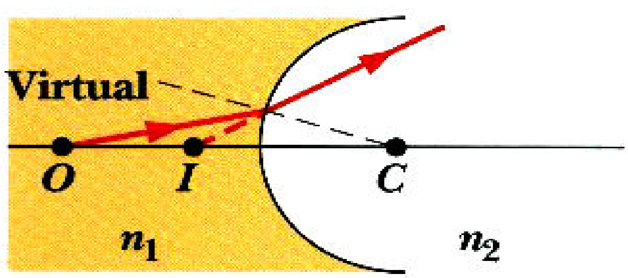
For spherically refracting **concave surfaces**, virtual images are always formed irrespective of object location. Refraction is always away from the central axis.



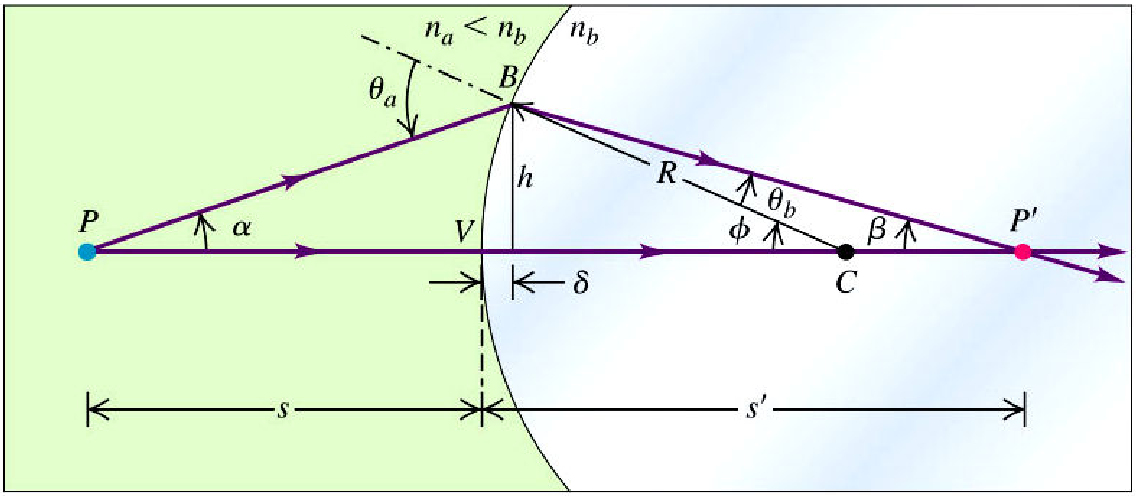
Locating objects **inside the medium of higher refraction index** will similarly produce real or virtual images depending on object proximity to a **concave lens surface**.



For a **convex lens** surface and **objects inside** the medium of higher index of refraction, virtual images are formed independent of the object nearness to the lens surface.



A relationship similar to the mirror equation may be derived for the single-surface spherical lens:



Snell's Law at the front of the lens has $n_b Sin(\theta_b) = n_a Sin(\theta_a)$

A small angle approximation then has $n_b\theta_b \cong n_a\theta_a$

From geometry, $\theta_a = \alpha + \varphi$ and $\varphi = \theta_b + \beta$

Using the angles, $n_a\alpha + n_b\beta = (n_b - n_a)\phi$

$$\alpha = \frac{h}{p} \qquad \varphi = \frac{h}{R} \qquad \beta = \frac{h}{i}$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Sign conventions: **Object facing a convex lens ➔** $R\_positive$

Object facing a concave lens ➔ $R\_negative$

Image formed on the side opposite light origin➔ $s'\_positive$

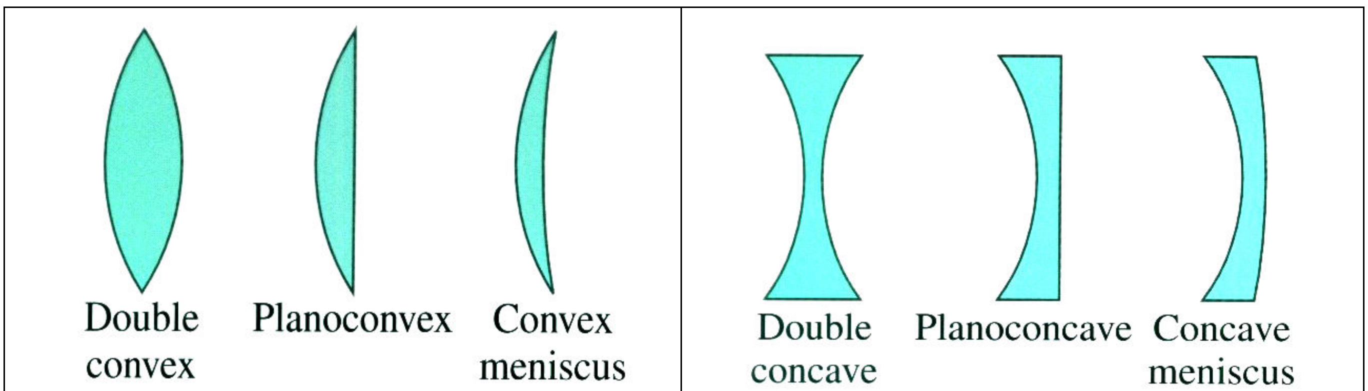
Images formed on same side light originates ➔ $s'\_negative$

The lateral magnification for a spherical refracting surface is: $$m = -\frac{n_a s'}{n_b s}.$$

Thin Lenses

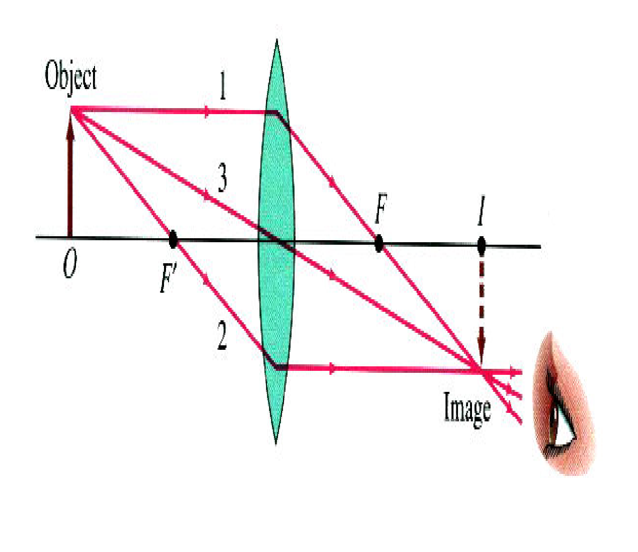
When the thickness of a lens is much smaller than its radii of curvature, the object distance and the image distance, then the lens is a **thin lens**.

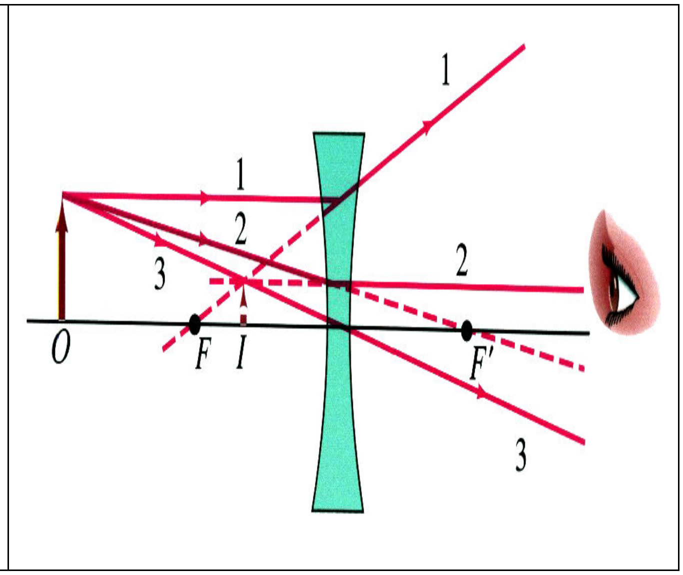
Simple lenses have two refracting surfaces and classified as converging or diverging.



(a) Converging lenses (b) Diverging lenses

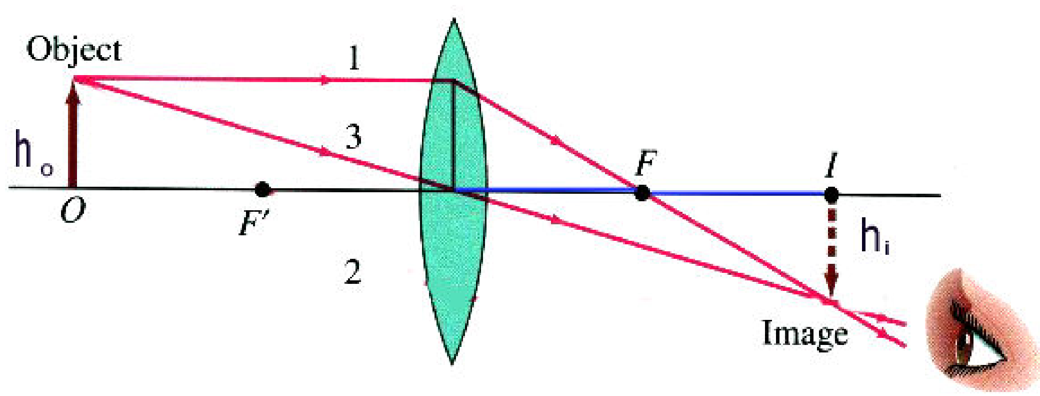
To begin, **ray-trace** through **thin symmetric** converging and diverging lenses and determine a table similar to that constructed for the spherical mirrors.





Lens Type	**Object Location**	**Image Location**	**Image Type**	**Image**	**Sign of** f	**Sign of** i	**Sign of** m
Diverging	**Anywhere**	**Same Side**	**Virtual**	**Upright**	**Negative**	**Negative**	**Positive**
Converging	**Inside** F	**Same Side**	**Virtual**	**Upright**	**Positive**	**Negative**	**Positive**
Converging	**Outside** F	**Opposite Side**	**Real**	**Inverted**	**Positive**	**Positive**	**Negative**

The **thin lens equation** and the **lens maker formula** are derived:



Thin Lens Equation

By similar triangles:

$$\frac{h_i}{h_o} = \frac{i}{p} \qquad \textbf{and} \qquad \frac{h_i}{h_o} = \frac{i-f}{f}$$

$$\frac{i}{p} = \frac{i-f}{f} \qquad \Rightarrow \qquad \frac{1}{p} = \frac{i-f}{if}$$

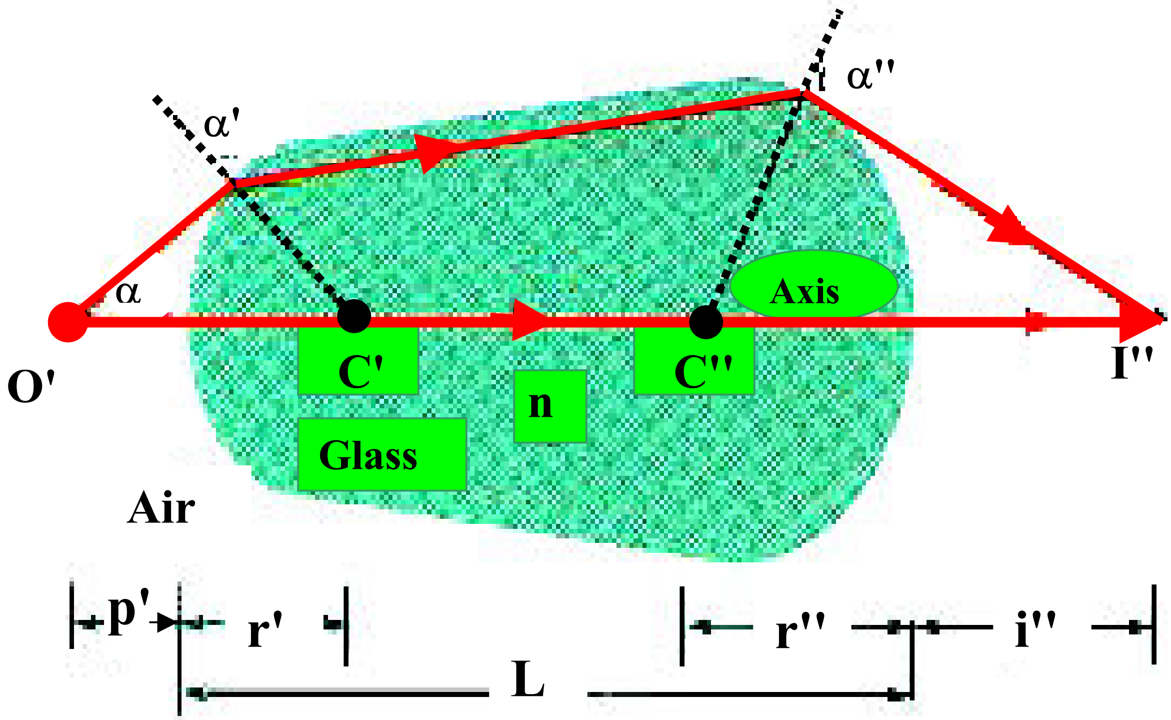
$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$$

For **diverging** lenses, f and i are entered as **negative** values.

Lensmaker formula

Consider a refraction occurring at each converging lens surface and apply the result found for refraction at a spherical surface:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$



First surface on left.

$$n_1 = 1 \qquad n_2 = n \qquad \frac{1}{p'} - \frac{n}{i'} = \frac{n-1}{r'}$$

Last surface on right.

$$n_1 = n \qquad n_2 = 1 \qquad \frac{n}{i'+L} + \frac{1}{i''} = \frac{1-n}{r''}$$

For thin lenses, $L \approx 0$ ➔ $$\frac{n}{i'} + \frac{1}{i''} = \frac{1-n}{r''}$$

Adding this to the result for the lens on the left,

$$\frac{1}{p'} + \frac{1}{i''} = \frac{n-1}{r'} - \frac{n-1}{r''}$$

Since p' is the initial object location, and i'' is the final image location:

$$\frac{1}{p} + \frac{1}{i} = (n-1) * \{\frac{1}{r'} - \frac{1}{r''}\}$$

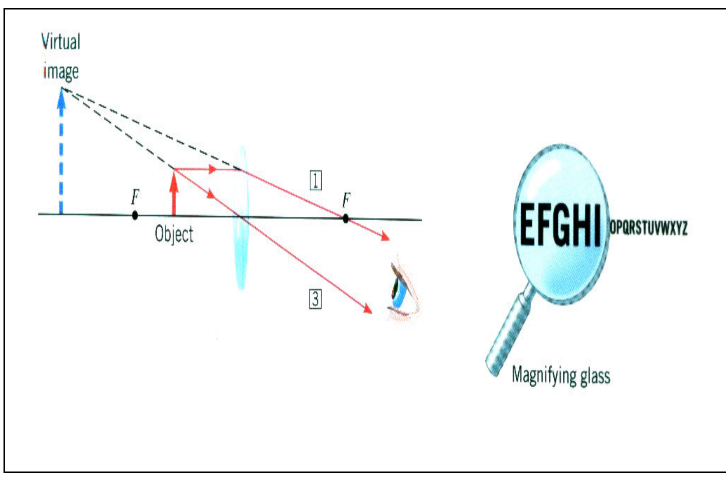
Lensmaker Formula

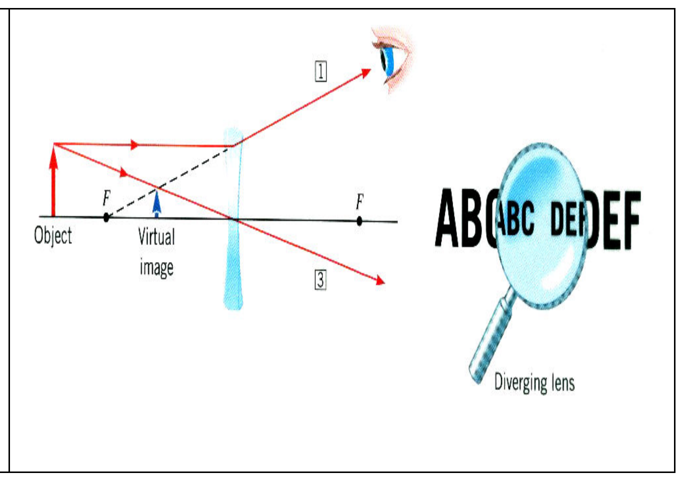
For submersion within a fluid, $n \rightarrow n / n_{fluid}$

Optical Instruments

Lenses are the principle components of optical instruments such as magnifiers, telescopes and microscopes. The goal is a well imaged, magnified view of the object be it a distant galaxy, a microscopic organism or the *fine print of a mortgage contract*:

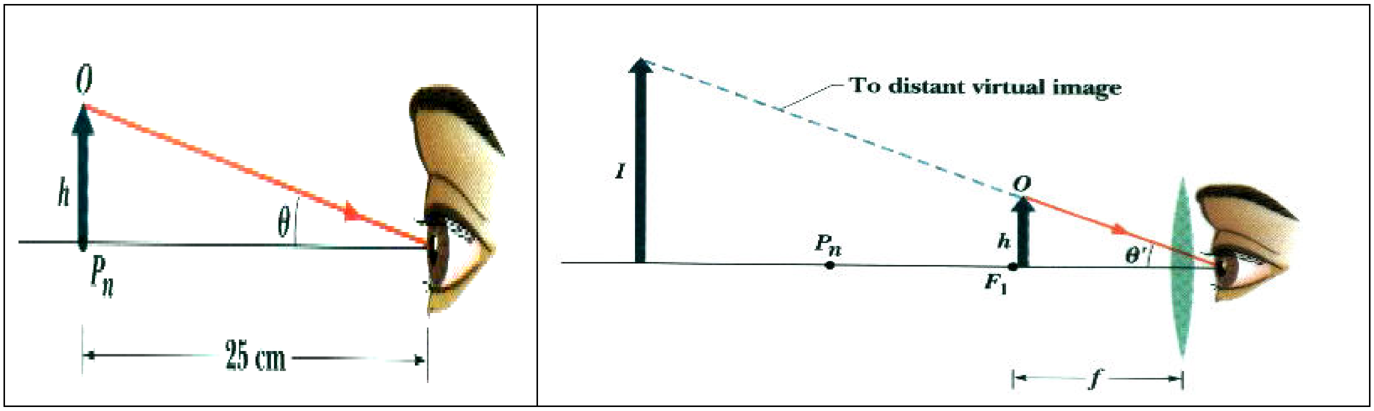
A view through each type of lens by itself is here:





The converging lens on the left is a simple magnifying glass. Note that with the object inside the focal length f, the image will be upright and on the side opposite the viewer.

The **angular magnification** of the glass is $m_\theta = \theta' / \theta$



$$\theta \approx h / 25cm$$

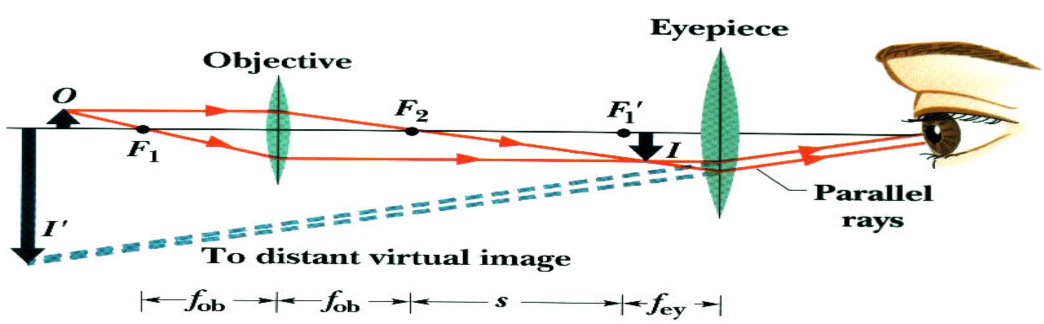
Assuming the human eye **near point** is **~25cm**.

$$\theta' \approx h / f$$

Small angle approx. and $m_\theta = 25cm / f$

Compound Microscopes

For microscopes, the problem is the magnification of very small objects close to the microscope **objective lens.**



The separation distance is adjusted so that when the objective image forms inverted just inside the eyepiece focal length, the eyepiece creates a magnified virtual image m_θ

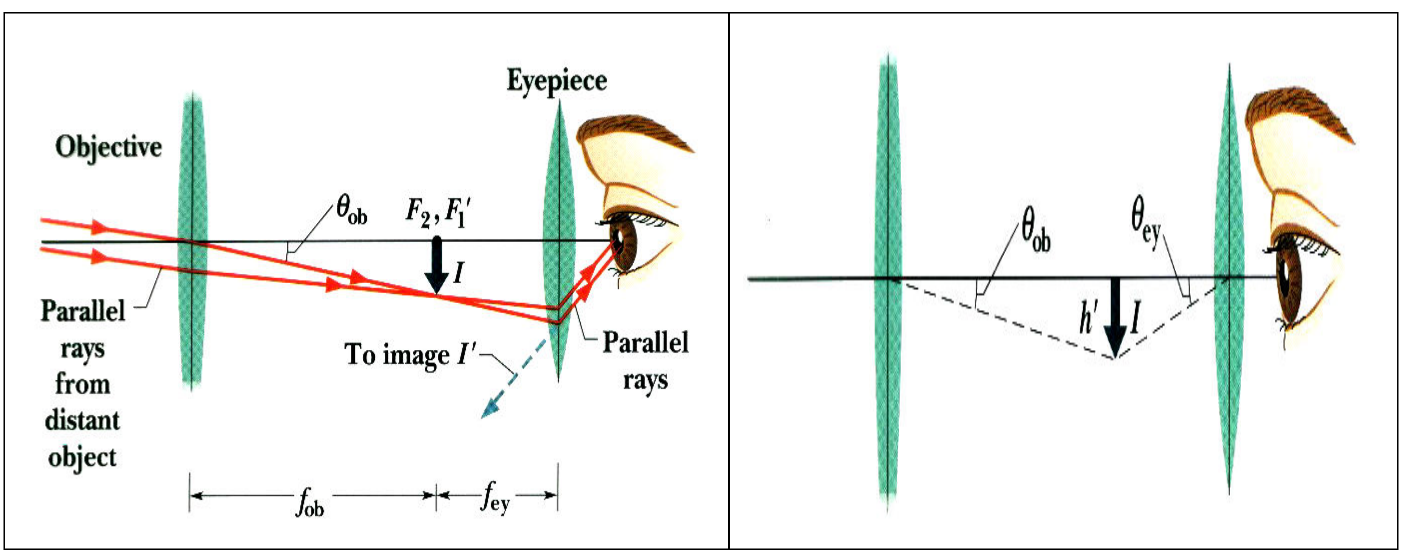
From the objective the lateral magnification is, $m = -\frac{i}{p} = -\frac{s}{f_{objective}}$ [$i \sim s$]

The compound lens magnification is the product of $m * m_\theta = M$

$$M = -\frac{s}{f_{objective}} \frac{25cm}{f_{eyepiece}}$$

Refracting Telescopes

For telescopes, the problem is the magnification of very large objects far removed from the telescope objective lens. Approximately parallel light rays at the objective.



The image from the objective forms at a **common focal point** for the objective and eyepiece lenses.

$$m_\theta = \frac{\theta_{eyepiece}}{\theta_{objective}} = \frac{h'}{f_{eyepiece}} \div \frac{h'}{f_{objective}}$$

$$m_\theta = -\,f_{objective} \big/ f_{eyepiece}$$

Inverted image in the eyepiece.

This is a ratio between the angle subtended by the object as imaged in the telescope to the angle subtended by the object when viewed without the telescope.

Field of View Resolving Power Light Gathering	Sagging Chromatic Aberration Seeing