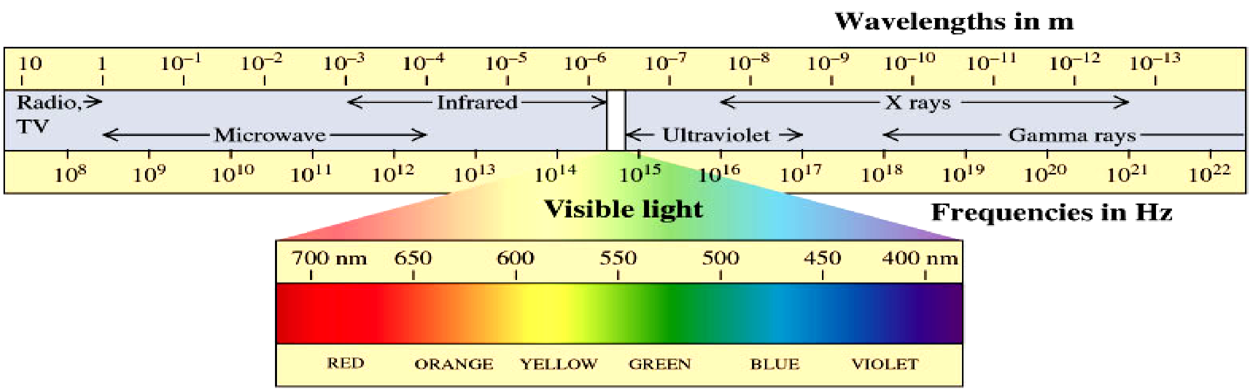
Electromagnetic Waves

Of all the incredible E&M phenomena described in the application of Maxwell's equations, perhaps the most enlightening is the existence of propagating waves of oscillating electric and magnetic fields.

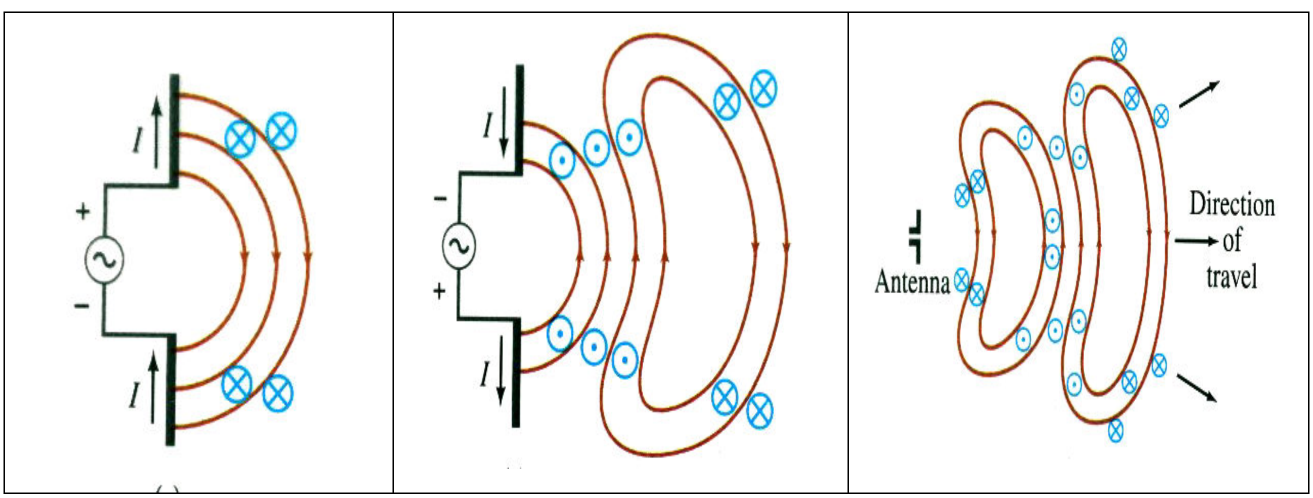
Electromagnetic radiation was first discovered in the visible portion of the **electromagnetic spectrum** for obvious reasons.

The spectrum extends continuously for several orders of magnitude beyond the visible range. Extremely Low Frequency → Gamma Rays.

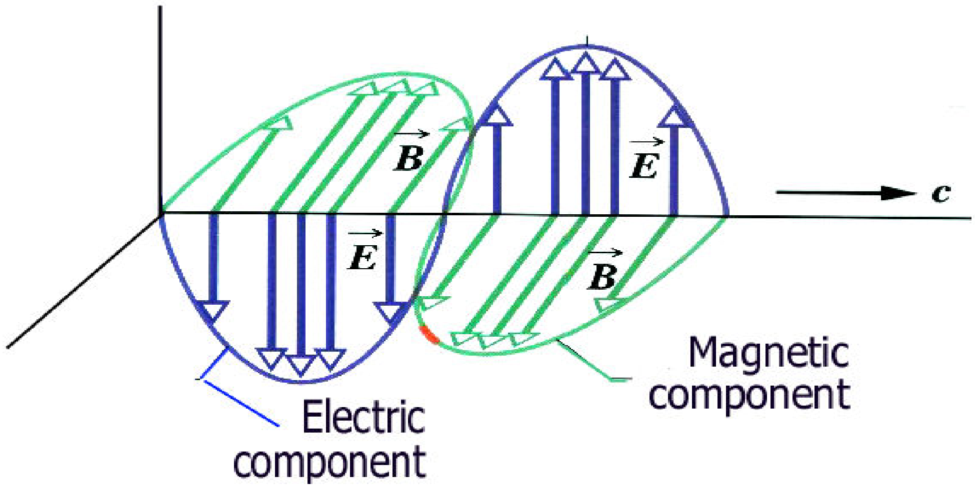


The **free space speed of light is a universal constant 299,792,458 m/s**.

In order to generate electromagnetic waves, charged particles may be accelerated in a dipole antenna as shown. The antenna current has a sinusoidal variation in magnitude and direction along the length of the antenna producing an oscillating electric dipole moment.



Recalling the field is everywhere directed tangent to the field lines, electromagnetic wave propagation is found to be a **transverse wave**.



1) $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation.

2) $\vec{E}$ is always perpendicular to $\vec{B}$.

3) $\vec{E} \times \vec{B}$ gives the direction of propagation.

4) Since the source produces an electric dipole moment sinusoidal in time, both $\vec{E}$ and $\vec{B}$ are sinusoidal with the same frequency and in phase.

$$E = E_m Sin(kx - \omega t)$$

$$B = B_m Sin(kx - \omega t)$$

$$k = 2\pi\lambda^{-1} \qquad \omega = 2\pi / T$$

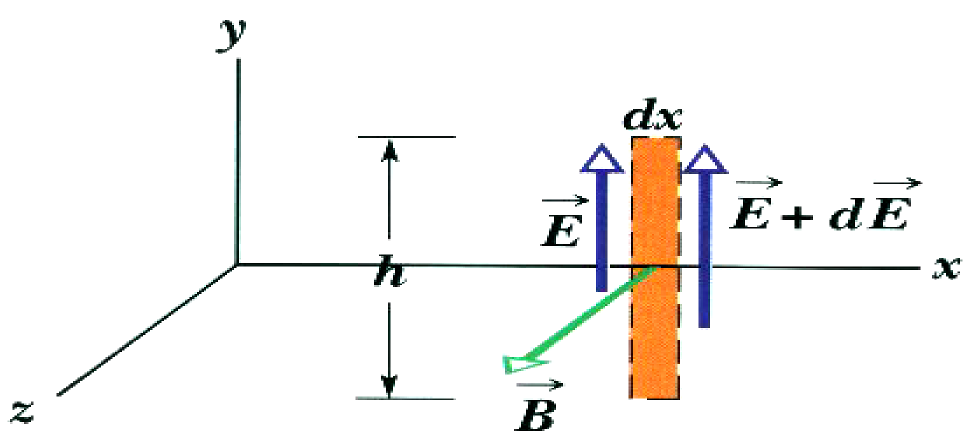
5) $\vec{E}$ and $\vec{B}$ continuously create one another via the electromagnetic laws of induction: time variations of $\vec{B}$ ➔ $\vec{E}$ by Faraday's Law and time variations of $\vec{E}$ ➔ $\vec{B}$ from Maxwell's Law.

6) Energy in the electromagnetic fields is proportional to $E_m{}^2$ and $B_m{}^2$

7) $\vec{E}$ & $\vec{B}$ magnitudes fall like r^{-1} and intensity falls off as r^{-2}

Induced Electromagnetic Waves

For our propagating wave, the Faraday induction is as follows:



$$E = E_m Sin(kx - \omega t) \qquad B = B_m Sin(kx - \omega t)$$

Take a counterclockwise path integration loop of height **h** and width **dx**. The changing magnetic flux in this region (decreasing field) of space induces an electric field as:

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$$

LHS is zero along the horizontal segments of the path integral so:

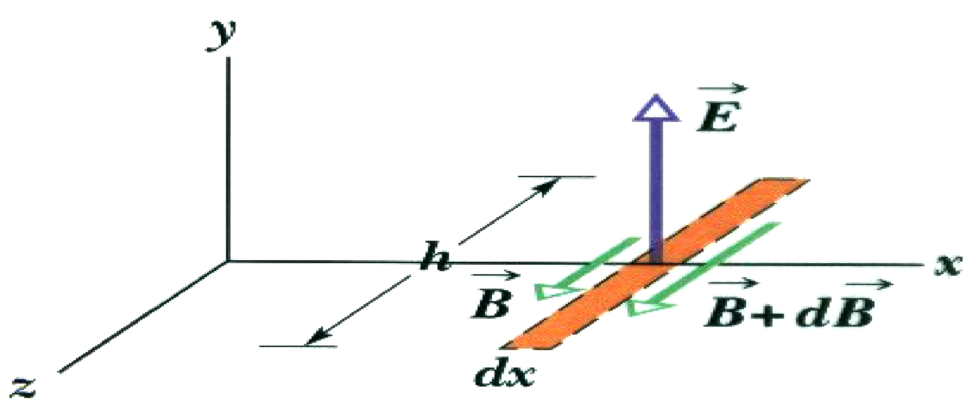
$$(E + dE)h - Eh = -\frac{d}{dt} Bhdx$$

$$hdE = -\frac{d}{dt} Bhdx \qquad \Rightarrow \qquad \frac{dE}{dx} = -\frac{dB}{dt}$$

$$kE_m Cos(kx - \omega t) = \omega B_m Cos(kx - \omega t)$$

From which results $\dfrac{E_m}{B_m} = \dfrac{\omega}{k} = \dfrac{2\pi f}{2\pi/\lambda} = c$

The Maxwell induction is:



Taking a counterclockwise path integration loop of height **h** and width **dx**, the changing electric flux in this region (decreasing field) of space induces a magnetic field as:

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

LHS is zero along the horizontal segments of the path integral so:

$$Bh-(B+dB)h=\mu_0\varepsilon_0\frac{d}{dt}Ehdx$$

$$-hdB=\mu_0\varepsilon_0\frac{d}{dt}Ehdx \qquad \Rightarrow \qquad -\frac{dB}{dx}=\mu_0\varepsilon_0\frac{dE}{dt}$$

$$-kB_m Cos(kx-\omega t)=-\mu_0\varepsilon_0\omega E_m Cos(kx-\omega t)$$

From which $\dfrac{E_m}{B_m}=\dfrac{1}{\mu_0\varepsilon_0 c}=c \qquad \Rightarrow \qquad c=\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

For electromagnetic waves propagating in medium characterized by non-free space values of permittivity and permeability, the speed of propagation within the medium is:

$$v=\frac{1}{\sqrt{\varepsilon\mu}}=\frac{1}{\sqrt{K\varepsilon_0 K_m\mu_0}}\cong\frac{c}{\sqrt{K}}$$

$$K_m\cong 1 \qquad non-ferromagnetic$$

The medium **index of refraction** is $n=\dfrac{c}{v}=\sqrt{KK_m}$

Poynting Vector

The rate at which energy is transported by the electromagnetic wave across a unit area at an **instance in time** is the **Poynting Vector**: $\vec{S}$

$$\vec{S} = \frac{1}{\mu_0}\{\vec{E} \times \vec{B}\} \qquad Units = \frac{Power}{Area} = \frac{W}{m^2}$$

The direction of $\vec{S}$ gives the direction of travel and the direction of energy transport.

The **instantaneous energy flow rate** is the Poynting Vector magnitude:

$$S = \frac{1}{\mu_0} EB = \frac{E^2}{\mu_0 c}$$

Averaged over time,

$$S_{Avg} = \frac{E_m{}^2 \overline{Sin^2(kx - \omega t)}}{\mu_0 c} = \frac{E_m{}^2}{2\mu_0 c} = \frac{1}{\mu_0 c} E^2{}_{RMS}$$

The average of the Poynting Vector is the electromagnetic wave **intensity.**

$$I = S_{Avg} = \frac{1}{\mu_0 c} E^2{}_{RMS}$$

Recall $I = P/4\pi r^2$ for an isotropic spherical source, implying $E \sim 1/r$

Since $B = E/c$ the magnetic field also has $B \sim 1/r$

Radiation Pressure

Electromagnetic waves have **linear momentum** in the direction of propagation that may be transferred to any object upon which the wave is incident.

If the incident wave is absorbed, then

$$\Delta P = \frac{\Delta U}{c} \qquad \Delta U = energy\_absorbed$$

For a reflected wave, the transfer to the object is

$$\Delta P = \frac{2\Delta U}{c} \qquad \Delta U = energy\_absorbed$$

The force on the absorbing object is:

$$F = \frac{\Delta P}{\Delta t} = \frac{\Delta U}{c\Delta t} = \frac{1}{c} * IA$$

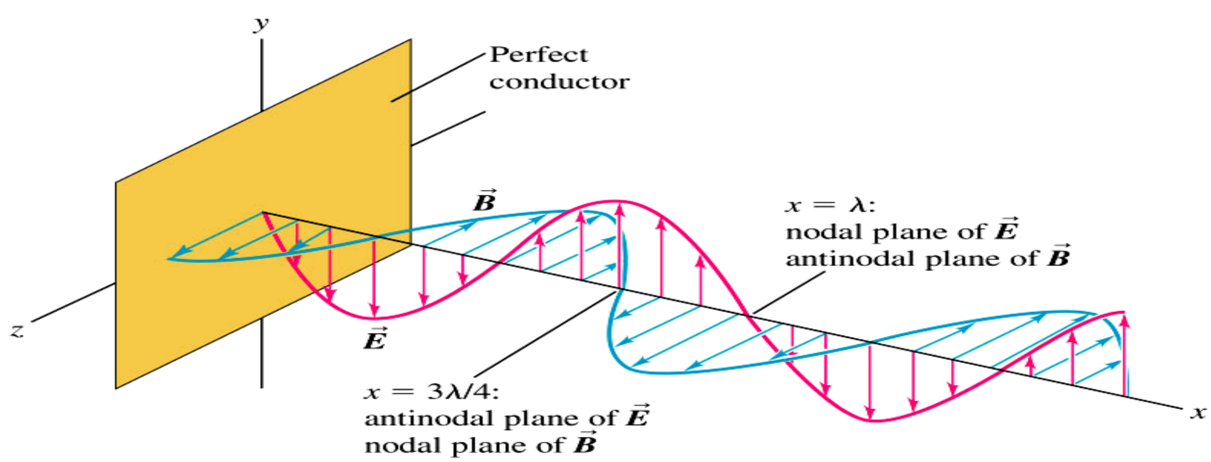
The radiation pressure associated with this force is $P_r = F/A = I/c$

For the reflecting object, $P_r = F/A = 2I/c$

Optical Tweezers and Ion Tails vs. Dust Tails.

Standing Electromagnetic Waves

At the surface of an ideal conductor, the condition of zero electric field inside the conductor requires that an electric field node exist there for any incident electromagnetic waves. Given that incident electromagnetic waves have an electric field varying in time, a superposition of this electric field with the reflected field generated at the surface forms this node and also **standing electromagnetic waves** for particular **cavity** dimensions.



For the coordinates shown, the superpositions are:

$$E_y(x,t) = E_{\max}\left[Cos(kx+\omega t) - Cos(kx-\omega t)\right]$$

$$B_z(x,t) = B_{\max}\left[-Cos(kx+\omega t) - Cos(kx-\omega t)\right]$$

$$E_y(x,t) = -2E_{\max} Sin(kx) Sin(\omega t)$$

$$B_z(x,t) = -2B_{\max} Cos(kx) Cos(\omega t)$$

Zero electric field for all time is observed at $kx = n\pi$

Zero magnetic field for all time is observed at $kx = (2n+1)\dfrac{\pi}{2}$

Note that the nodal planes for either the electric or magnetic fields correspond to the antinodal planes of the magnetic or electric fields respectively.

Normal modes within a cavity of two conducting surfaces separated by distance **L** are determined by the boundary condition requiring nodes at each conductor:

$$L = n\frac{\lambda}{2} \qquad\qquad f_n = \frac{c}{\lambda_n} = n\frac{c}{2L}$$