## Alternating Current and AC Circuits

For a sinusoidal oscillating voltage, the current in a circuit will be **alternating current**.

AC current / voltage transmissions have the advantage of not being as costly in terms of dissipative resistive loss. Since power loss is $P = IV = I^2R$ and the average AC power includes averaging over a Sine squared function that gives a factor of $\frac{1}{2}$

In addition, AC transmission at high voltage (~100's kV) and low current is possible since transforming high voltages with step-down transformers for consumer use is easier with AC. This limits resistive loss.

$$V = V_0 Cos(\omega t)$$ Where $$\omega/2\pi = 60Hz$$

$$I = I_0 Cos(\omega t)$$

$V_0$ and $I_0$ Are **Peak values** of the voltage and current.

$$\overline{P} = RI_0{}^2\,\overline{Sin^2\omega t} = \frac{1}{2}I_0{}^2R$$

Measured at the wall with a **DVM** are the **RMS** voltage and current:

$$V_{rms} = \sqrt{\overline{V^2}} \qquad I_{rms} = \sqrt{\overline{I^2}}$$

$$\overline{P} = I_{rms}V_{rms} = \frac{1}{2}I_0V_0$$

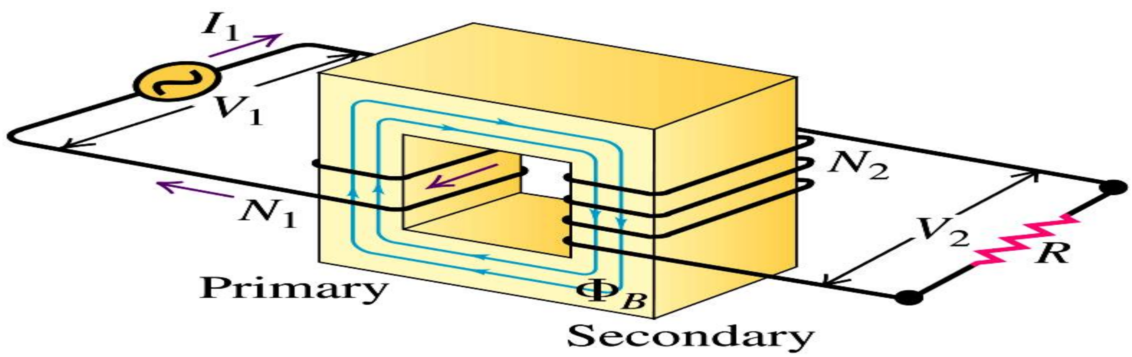
➔

$$I_{rms} = {I_0}/{\sqrt{2}} \qquad V_{rms} = {V_0}/{\sqrt{2}}$$

**Transformers**

Transformers are used to increase or decrease voltages to an appropriate value for a given application.

**Primary** and **secondary** coils with differing numbers of turns are **flux-linked** such that voltages may be either **stepped-up** or **stepped-down**:






$$V_2 = N_2 * \frac{\Delta\Phi_B}{\Delta t} = N_2 * \frac{V_1}{N_1}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

For 100% efficient transformers, power input equals power output. If the voltage is stepped-up the current in the secondary must decrease and vice-versa.

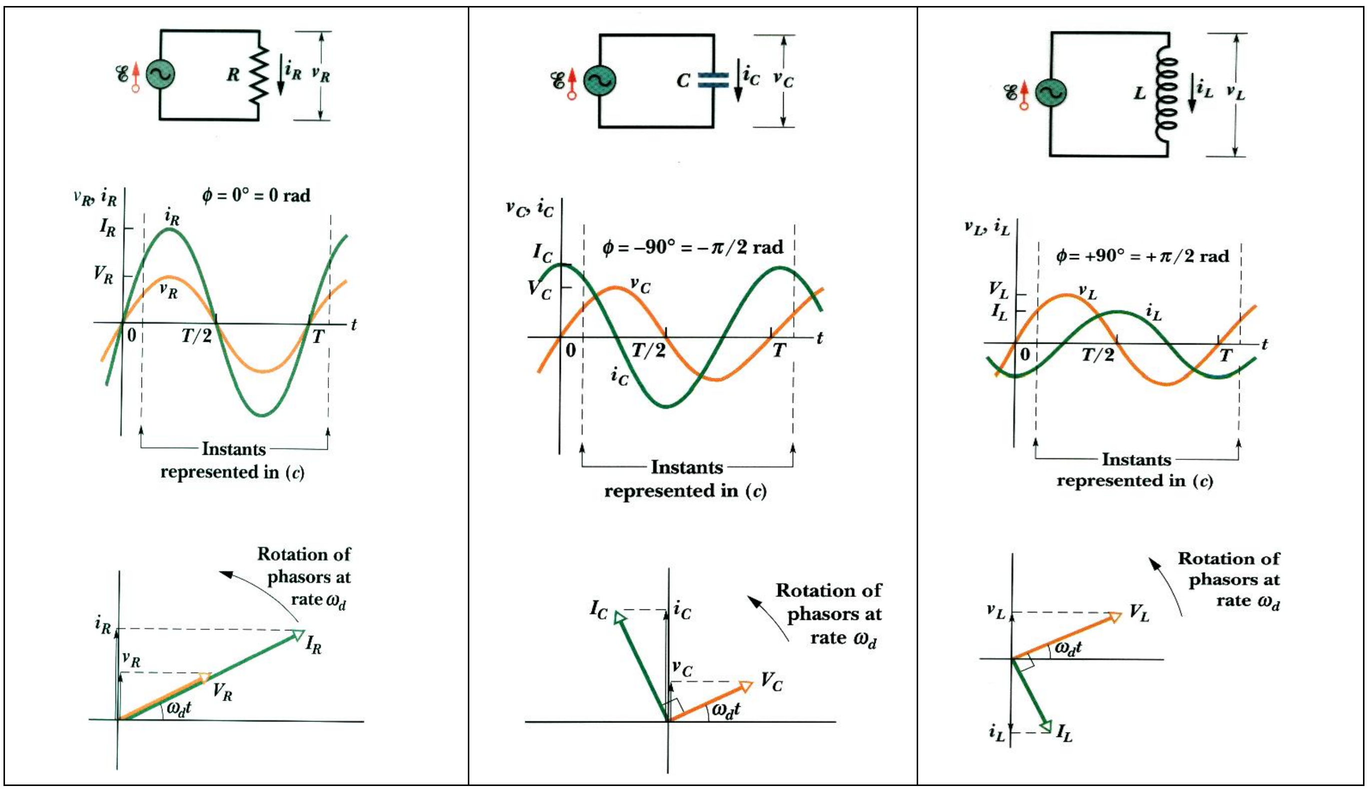
$$I_1 V_1 = I_2 V_2$$

## AC and LRC Circuit Elements

Since any input signal may be **Fourier analyzed** into a series summation of sine and cosine inputs, the response of a circuit to a general AC signal is important.

Depending on the combination of circuit elements **R, L, and/or C**, the driving emf and circuit current will in general be out of phase with each other.

$$E = E_m Cos(\omega_d t + \phi) \qquad i = ICos(\omega_d t)$$




**Phasor projections along the vertical ➔current / voltage at time = t**

## Resistive Load

For the above emf / resistor only circuit:

$$E - v_R = 0$$

$$\rightarrow v_R = E_m Cos(\omega_d t) = V_R Cos(\omega_d t)$$

$$i_R = \frac{V_R}{R} Cos(\omega_d t) = I_R Cos(\omega_d t)$$ ➔ $$V_R = IR$$

**Current and voltage are in phase**.

## Capacitive Load

Using the AC emf with a capacitor:

$$i_C = ICos(\omega_d t) \qquad q_C = \int idt = \frac{I}{\omega_d} Sin(\omega_d t)$$

$$q = Cv \Longrightarrow v_C = \frac{I}{\omega_d C} Sin(\omega_d t)$$

**Defining the capacitive reactance** $$X_C = \frac{1}{\omega_d C}$$

$$v_C = V_c Cos(\omega_d t - \frac{\pi}{2})$$ ➔ **Voltage lags current by 90 degrees**

Then since $$v_C = V_c Cos(\omega_d t + \phi) \qquad \phi = -\pi/2$$

$$V_C = I_C X_C$$

## Inductive Load

AC source and inductor only:

$$v_L = -L\frac{di}{dt} = I\omega L Sin(\omega_d t)$$

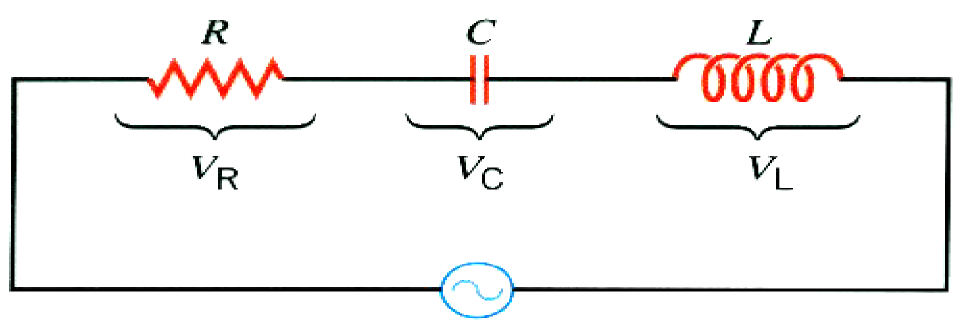
Defining the **Inductive reactance** as $X_L = \omega_d L$

$$v_L = I\omega L Cos(\omega_d t + \frac{\pi}{2})$$ ➔**Voltage leads current by 90 degrees**

Then since $v_L = V_L Cos(\omega_d t + \phi)$ ➔ $\phi = +\pi/2$

And $V_L = I_L X_L$

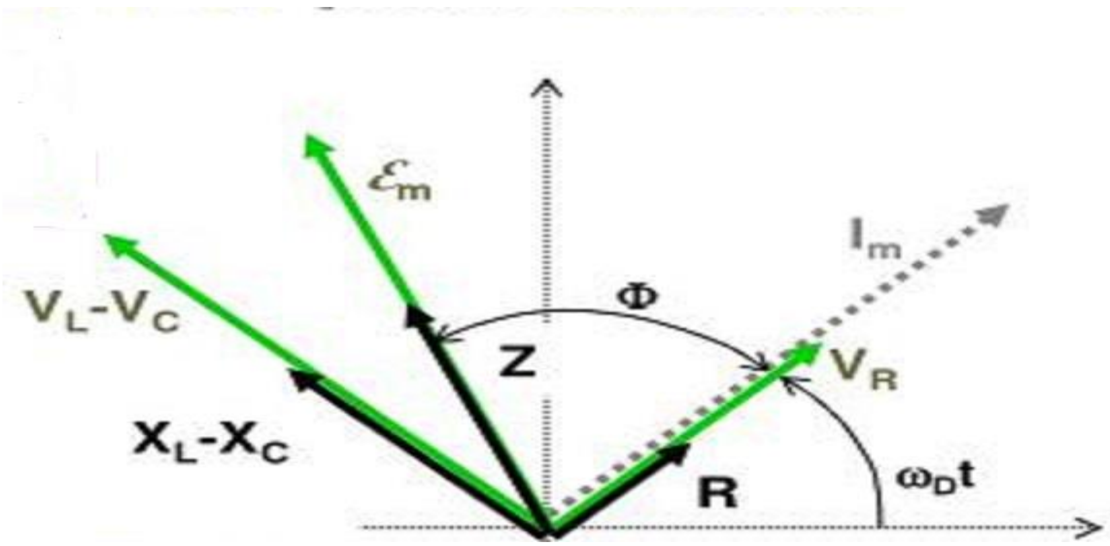
## Series RLC Circuits



In general: $$E = E_m Cos(\omega_d t + \phi) \qquad i = ICos(\omega_d t)$$

$I$ and $\phi$ To be determined.

The Phasor algebra looks like:




From Kirchhoff's Rule, $$E_m - V_R - V_L - V_C = 0$$

The current and $V_R$ are in phase and the phase angle between current and emf is $\phi$ .

$$E_m{}^2 = V_R{}^2 + (V_L - V_C)^2 = I^2 R^2 + I^2 (X_L - X_C)^2$$

$$I = \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The **impedance** is: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$I = E_m / Z = \frac{E_m}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}$$

From the figure, the phase angle $\phi$ is:

$$Tan(\phi) = \frac{V_L - |V_C|}{V_R} = \frac{X_L - X_C}{R}$$

Three cases are as follows:

1) **Inductive circuit** $\phi > 0$ **Voltage leads current**
2) **Capacitive circuit** $\phi < 0$ **Voltage lags current**
3) **Resonance state** $\phi = 0$ **Voltage and current are in phase.**

Starting the emf at low frequency and scanning to higher values of $\omega_d$ will produce an observable phase shift in $i$ relative to $E$ as the circuit moves from a capacitive circuit to an inductive circuit across the resonance point at $\omega_d = 1/\sqrt{LC}$

**Power**

For the RLC circuit, the phase difference between voltage and current in both the inductor and capacitor means that the average power transferred to these elements is zero and the average power dissipated by the circuit is due solely to the resistor:

$$\overline{P} = \overline{i^2 R} = I^2 R \overline{Sin^2(\omega_d t - \phi)} = \frac{1}{2} I^2 R = \left(\frac{I}{\sqrt{2}}\right)^2 R = (I_{RMS})^2 R$$

$\overline{P} = (I_{RMS})^2 R$ Using RMS current ➔ power is calculated just like DC.

$$\overline{P} = (I_{RMS})^2 R = \frac{E_{RMS}}{Z} I_{RMS} R = E_{RMS} I_{RMS} \frac{R}{Z}$$

$$\frac{R}{Z} = Cos(\phi) = Power\_factor$$

$$R = Z \qquad \Rightarrow \qquad \phi = 0 \qquad i.e., \qquad resonance$$

Maximum power is transferred to $R$ at resonance.

For inductors or capacitors, $\phi = \pm 90^0$ and no average power is dissipated.

## Resonance

Driving an RLC circuit at **resonance condition** produces output current and voltage amplitudes that are maximal. The condition for resonance is: $\omega_{Driving} = \omega_0$

| | |
|---|---|
| $I_{rms} = \dfrac{V_{rms}}{Z}\dfrac{V_{rms}}{\sqrt{R^2 + (\omega L - \dfrac{1}{\omega C})^2}}$ | $Gain = \left\lvert \dfrac{V_{out}}{V_{in}} \right\rvert = \dfrac{R}{\sqrt{R^2 + (\omega L - \dfrac{1}{\omega C})^2}}$ |

Notice as the damping $R$ is reduced the resonant amplitude peaks become larger and have narrower half-width. The series RLC is a useful bandwidth filter circuit near the resonance frequency.