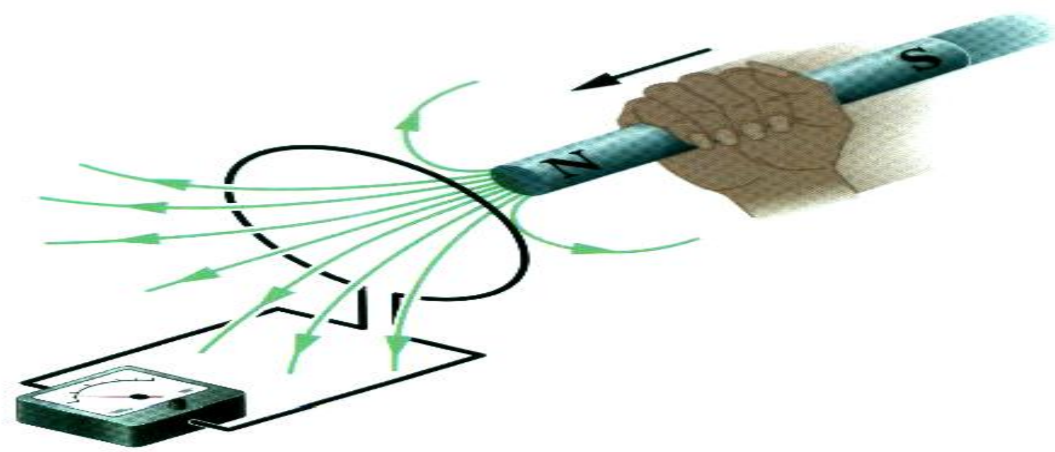
Induction

In 1830-1831, **Joseph Henry & Michael Faraday** discovered **electromagnetic induction**. Induction requires **time varying magnetic fields** and is the subject of another of Maxwell's Equations.

Modifying Ampere's Law to include the possibility of **time varying electric fields** gives the fourth Maxwell's Equations.

Faraday's Law

Changing magnetic flux in a conducting loop produces an **induced emf** in the conducting loop that depends on the time rate of change of the magnetic flux.



For the conducting loop, find the magnetic flux and its time rate of change:

$$\Phi = \int \vec{B} \cdot \hat{n} dA = \int BCos(\theta) dA$$

SI unit of Φ is the Weber 1W = 1Tm^2

Three ways by which the flux may vary in time are:

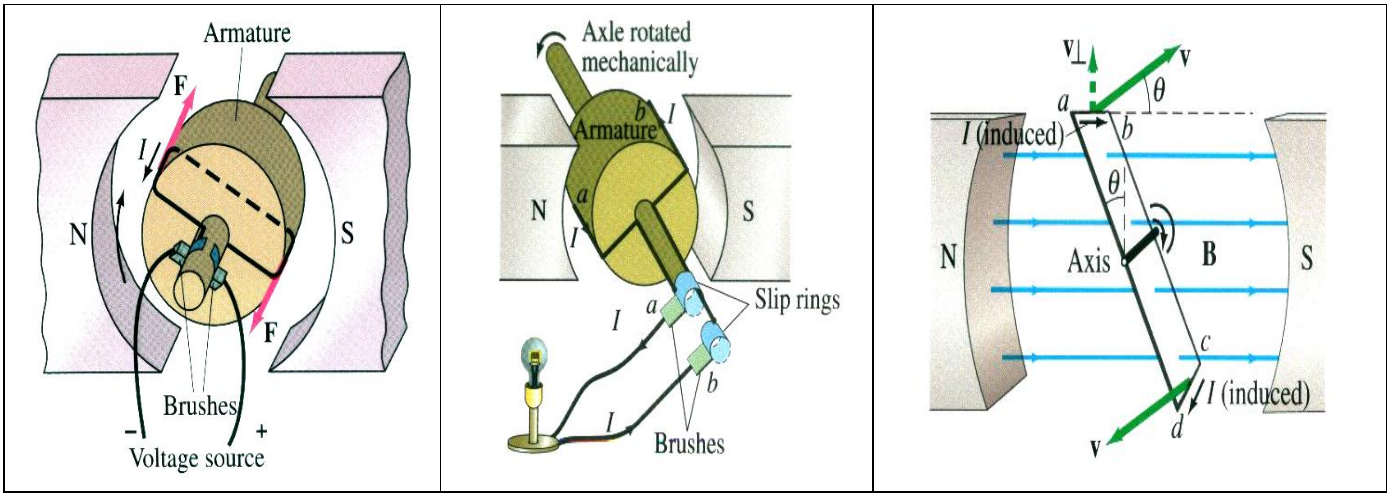
1) Amount of magnetic field changes

2) Loop cross-section area changes

3) Changes in the relative orientation of $\vec{B}\_to\_\hat{n}$

Method 3) is used in an **electric generator** with input mechanical energy.

Effectively the opposite of an electric motor, the changing orientation of the loop $\hat{n}$ relative to $\vec{B}$ results in a generated emf via induction in the conducting loop.



For a changing magnetic flux, the induced emf in the conducting loop is:

$$Emf = -\frac{d\Phi}{dt}$$

In an **AC generator**, if the loop rotates at a constant angular velocity ω :

$$Emf = -\frac{d\Phi}{dt} = -BA\frac{d}{dt}\left(Cos(\omega t)\right) = BA\omega Sin(\omega t)$$

Using a coil with N tightly packed loops,

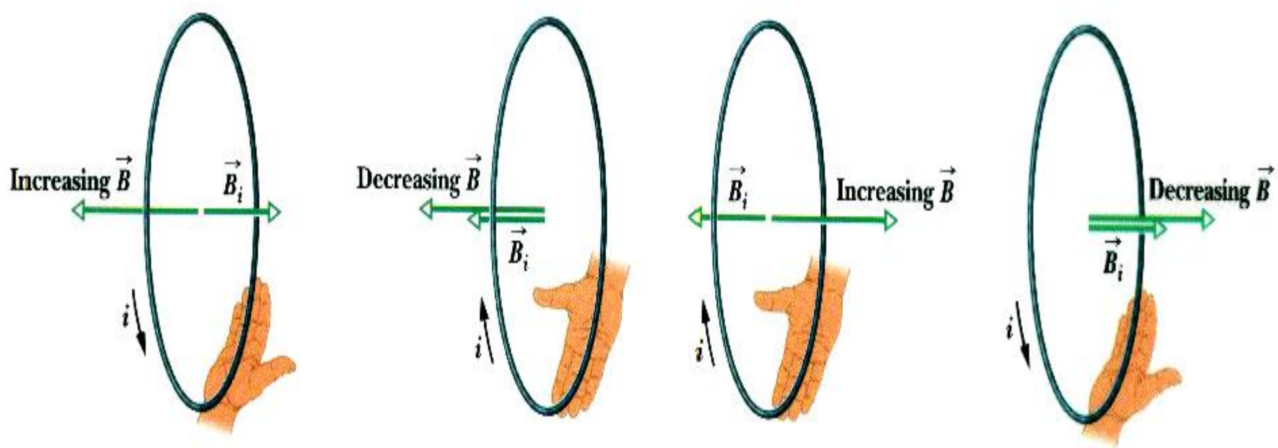
$$Emf = NBA\omega Sin(\omega t) = E_0 Sin(\omega t)$$

Note the **minus sign** in the induced emf: $Emf = -\frac{d\Phi}{dt}$

The minus sign implies the direction of flow for the electric current resulting from the induced emf. This is described by **Lenz's Law**.

Lenz's Law

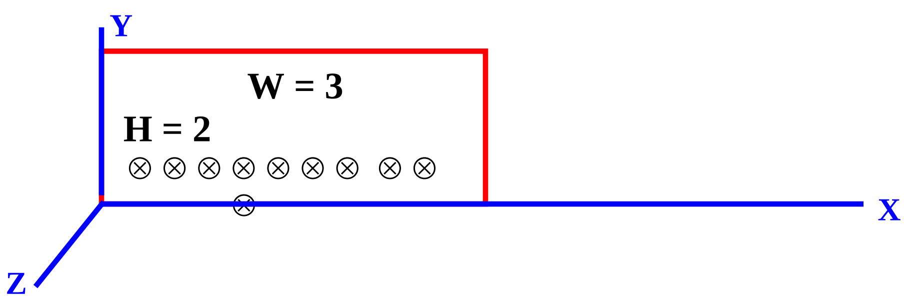
Lenz's Law is: The direction of current corresponding to the induced emf is such that the magnetic field resulting from this current opposes the change in magnetic flux that induces the current.



A changing magnetic field flux ➔ Induced current direction is such that its $\vec{B}$ field direction **opposes** flux field changes.

E.g., A time varying $\vec{B}$ established in area **HxW** defined by a conducting wire loop

$\vec{B} = 4t^2x^2(-\hat{k})$ Find the induced emf at time $t = .10s$



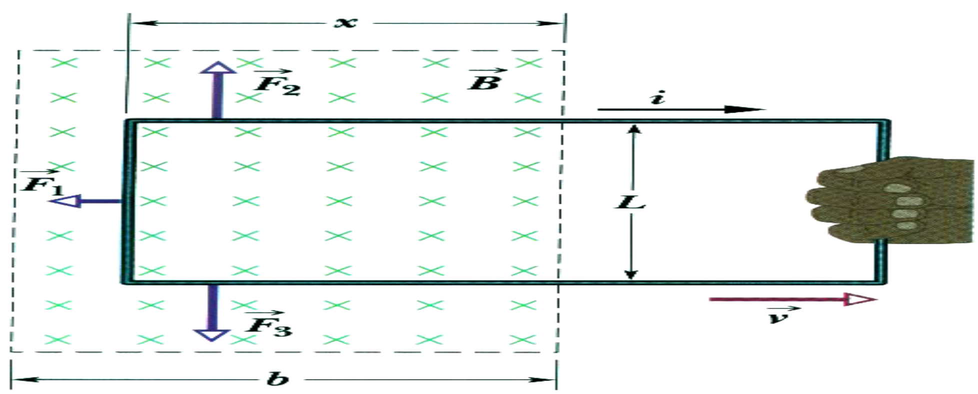
$$\Phi = \int \vec{B} \cdot \hat{n} dA = \int 4t^2 x^2 Cos(\pi) H dx = -4Ht^2 \{\frac{x^3}{3}\}\Big|_0^3 = -72t^2$$

$$Emf = -\frac{d\Phi}{dt} = 144t \qquad At\_t = 0.1s \qquad Emf = 14.4V$$

Increasing $\vec{B}$ into the page implies current will be counterclockwise.

Motional Emf

For a circuit or loop of wire moving within a magnetic field, changing magnetic flux can induce an emf because of the conductor motion.



As the loop is removed from the region of magnetic field, a '__________' current is induced. Moreover, since this is a current carrying wire in a magnetic field, a force $F = BiL$ is present on each of the segments within the field.

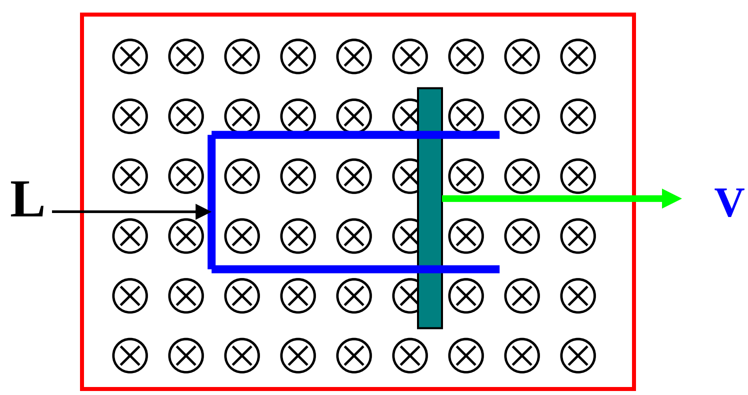
Along the top and bottom portions of the wire, these forces cancel. However, the external agent must supply power $P = \vec{F} \cdot \vec{v}$ to move the loop within the magnetic field since the force on the vertical segment of wire in the field is directed opposite to $\vec{v}$.

$$F_{Ext} = BiL = B * \frac{E}{R} * L = \frac{BL}{R} * \frac{d\Phi}{dt} = \frac{BL}{R} * \frac{d}{dt}(BXL) = \frac{B^2 L^2 v}{R}$$

$$P = FvCos(0) = \frac{B^2 L^2 v^2}{R}$$ Rate of doing work to move the loop at $\vec{v}$.

The resistive heat in the loop is also $P = i^2 R = \frac{E^2}{R} = \frac{B^2 L^2 v^2}{R}$

Fixing the loop in place, emf can be generated by moving a rail on a U-shaped conductor:

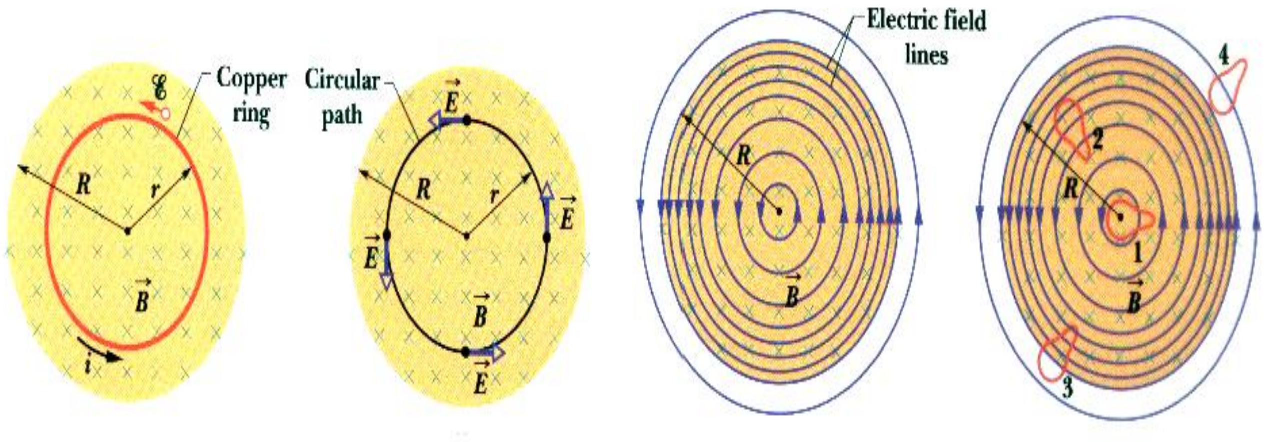


Electric Potential and Time Varying Magnetic Fields

Within a time varying magnetic field, the possibility of induced emf subverts Kirchhoff's loop rule:

$$\Delta V = \oint \vec{E} \cdot \vec{ds} \neq 0 \qquad Instead \qquad \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi}{dt}$$

Note electric potential has lost meaning when electric fields are the result of time varying magnetic fields. Forces associated with changing magnetic fields are **not** conservative.



Counter Emf, Counter Torque, and Eddy Current

In the case of an electric motor, as the driving current turns coils within the magnetic field there is an emf generated in the conducting loops that has current opposing the supply current. This is the **back emf** and depends on the rotation rate of the coils.

For the electric generator, an attached load draws current through the generator coils, which produces a **counter torque** at the axel.

Eddy currents form in general when a conductor moves through a region of magnetic field such that forces exist on the free electrons in that material. Eddy currents require energy that is extracted from the moving conductor thereby having a dissipating effect.

Ampere-Maxwell Law

Faraday's Law says a changing magnetic flux induces emf or, changing magnetic fields produce electric fields. A question then arising is, does an induction of magnetic field result from changing electric fields?

The answer is yes and is quantified in the fourth and last of Maxwell's equations called the **Ampere-Maxwell Law**:

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere's Law is regained if the electric flux is constant in time.

If $i_{enc} = 0$ but electric flux changes, then **Maxwell's Law of Induction** results.

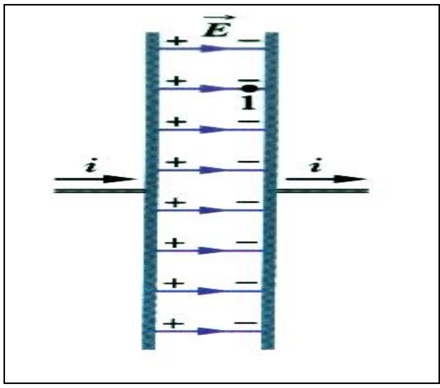
The 2nd term on the RHS is $\mu_0 * Displacement\_Current = \mu_0 i_d$

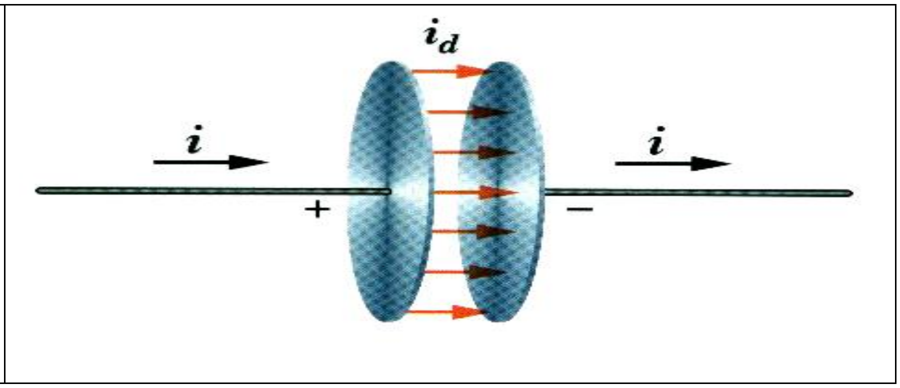
$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

This neither is a 'current' nor involves displacement but references the fictional ether.

The **displacement current** is not an actual current in the sense of charge flow, but the analogy is marginally supported through an example of a charging circular capacitor:

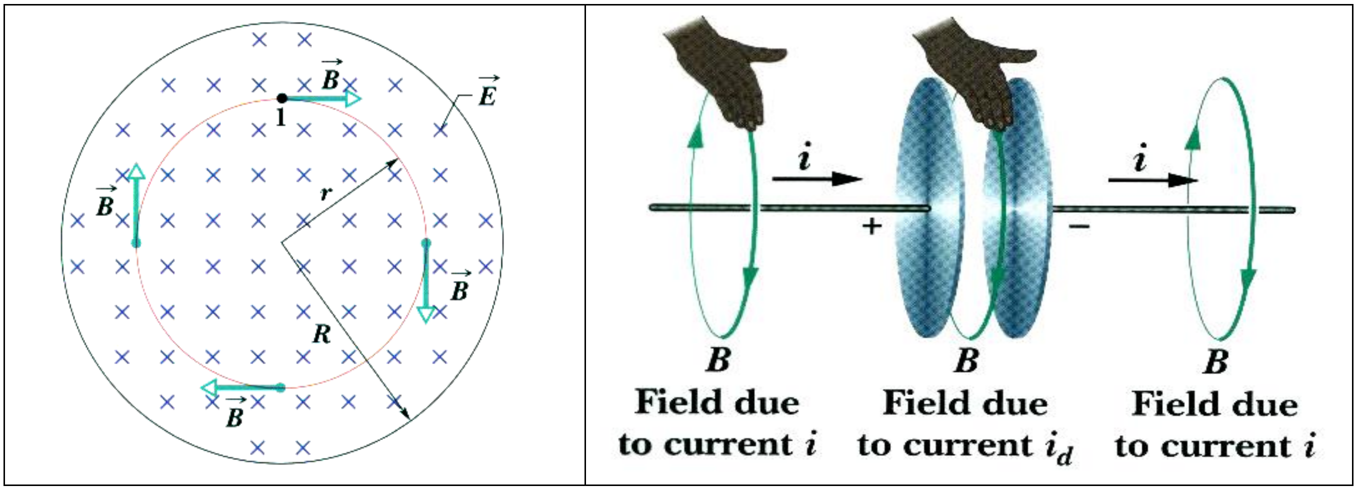
As a parallel plate capacitor attached to an emf is charged, the field in the gap between the two conducting plates increases from zero to **V/d**. This represents a changing $\mathbf{\Phi_E}$.





In the gap there is, say, a vacuum and therefore no conducting wires for a 'current' proper.

That is, $i_{enc} = 0$ in the gap region. The electric flux is changing and there are both electric and magnetic fields in the gap during the charging process of the capacitor.



As the capacitor charges, charge on the conducting plates is evaluated using Gauss's Law:

The **'real'** current is
$$i = \frac{dq}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

The **displacement** current is
$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} EA = i$$

Therefore, $i = i_d$ and displacement current is thus viewed as an extension of 'real' current through the capacitor gap region such that Ampere's law in this region gives:

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 i_{d,enc}$$

$$B \cdot 2\pi r = \mu_0 \frac{\pi r^2}{\pi R^2} i_d \quad \Rightarrow \quad B = \mu_0 \frac{r}{2\pi R^2} i_d$$

Inside

Maxwell's Equations in Integral Form for Free Space

Gauss's Law for Electric Fields	$\oint \vec{E} \cdot \hat{n} dA = {q_{enc}}/{\varepsilon_0}$
Gauss's Law for Magnetic Fields	$\oint \vec{B} \cdot \hat{n} dA = 0$
Faraday's Law of Induction	$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$
Ampere-Maxwell Law	$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

Superconductivity

Discovered in 1911 by **H.K. Onnes**, superconductivity is the loss of all electrical resistance by an element, inter-metallic alloy or compound below a certain **critical temperature Tc.**

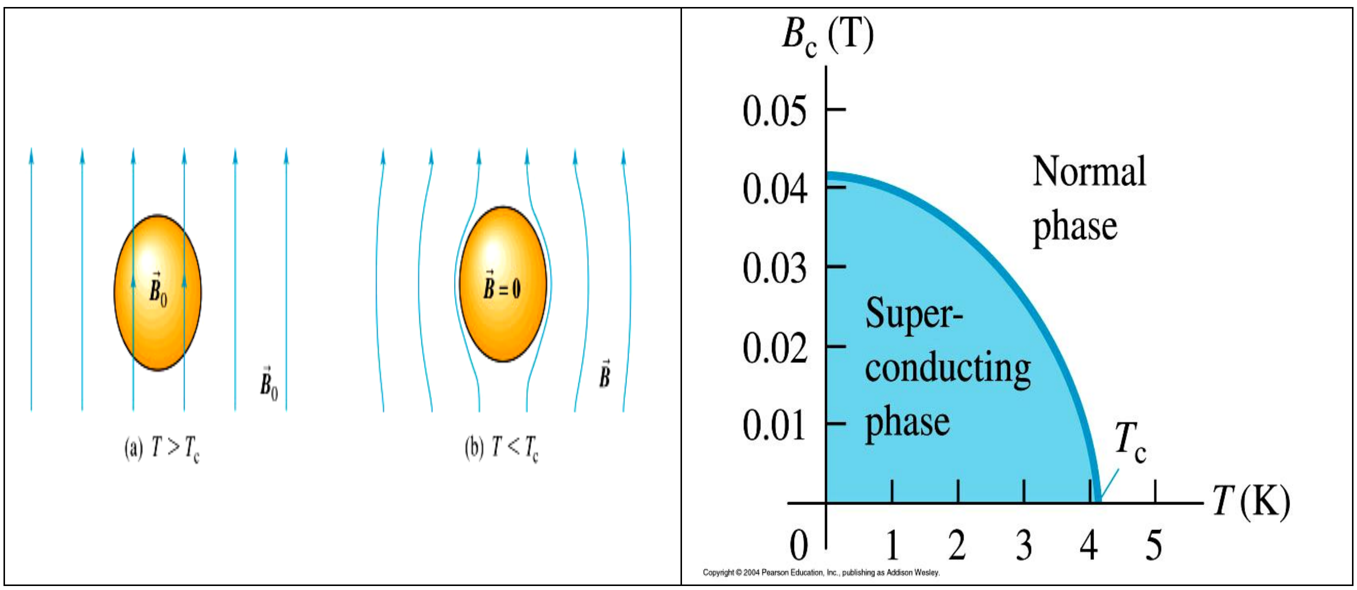
The highest **Tc** is 138 K seen in a ceramic material thallium-doped, mercuric-cuprate comprised of the elements Mercury, Thallium, Barium, Calcium, Copper and Oxygen

138 K is well above the 77 K liquid Nitrogen boiling point, which makes ceramic superconductors economically attractive. Current formability and strength issues represent technological barriers for High **Tc** superconductors.

Type 1 category superconductors are mainly metals that require liquid Helium temperatures around 4 K in order to establish "**Cooper pair**" formation and a superconducting transition. The formation is a **"phonon-mediated coupling"** involving sound packets generated by the flexing material lattice. The theory describing this type of superconductivity is **BCS.**

Lead (Pb)	**7.196 K**
Lanthanum (La)	**4.88 K**
Tantalum (Ta)	**4.47 K**
Mercury (Hg)	**4.15 K**
Tin (Sn)	**3.72 K**
Indium (In)	**3.41 K**
Thallium (Tl)	**2.38 K**
Rhenium (Re)	**1.697 K**
Protactinium (Pa)	**1.40 K**
Thorium (Th)	**1.38 K**
Aluminum (Al)	**1.175 K**
Gallium (Ga)	**1.083 K**
Molybdenum (Mo)	**0.915 K**
Zinc (Zn)	**0.85 K**

Type 1 superconductors will expel any external applied magnetic field from its interior while in a superconducting state (**Meissner Effect**), but may also be brought back into a normal state if the external magnetic field is greater than a critical value **Bc**.



Type 2 superconductors are metallic compounds, metal-oxide ceramics, and alloys. They achieve higher **Tc's** than **Type 1** superconductors that may be due to conduction from positively charged vacancies within the lattice called **Holes**.

Type 2 superconductors allow some penetration by an external magnetic field into its surface that produces thin filaments of normal phase material, oriented parallel to the applied field, around which currents circulate.

Type 2 superconductors have higher **Bc** making these more useful in applications where high fields are needed.

Two **Bc** exist for **Type 2** superconductors: **B_{c1}** is the field value at which external fields enter the material and **B_{c2}** is the field value when the material returns to a normal phase.

$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	**138 K**
$HgBa_2Ca_2Cu_3O_8$	**133-135 K**
$HgBa_2Ca_3Cu_4O_{10+}$	**125-126 K**
$HgBa_2Ca_{1-x}Sr_xCu_2O_{6+}$	**123-125 K**
$HgBa_2CuO_{4+}$	**94-98 K**