Equilibrium

For the static or equilibrium applications, require that all translation accelerations and rotational accelerations be identically zero:

$$\sum \tau_{Net\_Ext} = 0$$

$$\sum F_{Net\_Ext,X} = 0$$

$$\sum F_{Net\_Ext,Y} = 0$$

$$\sum F_{Net\_Ext,Z} = 0$$

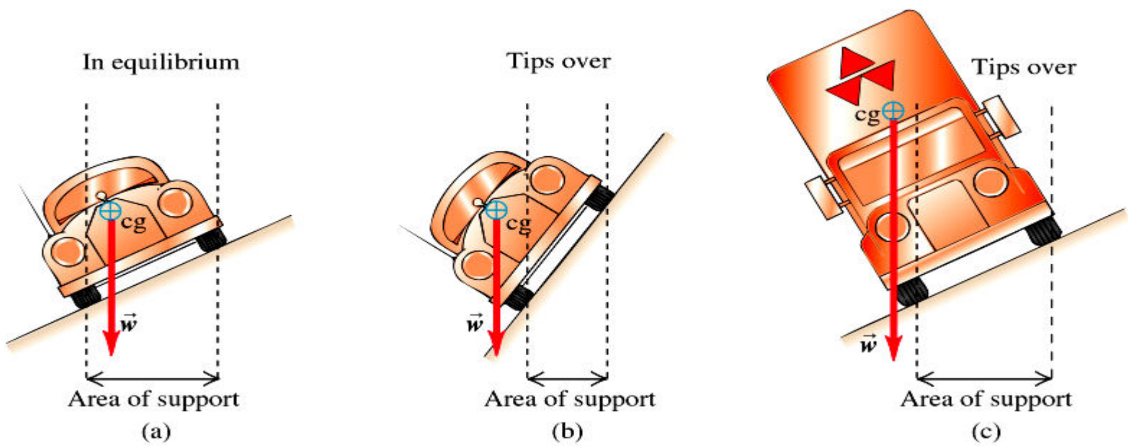
The constraints of **static equilibrium** supply us with the equations from which we may determine the conditions under which the system remains static.

Notice that a system may be in motion with a uniform velocity and still satisfy these conditions. This is **non-static equilibrium**.

Balance and Stability

The issue of **balance and stability** principally depends on the relative location of the objects center of gravity $\vec{R}_{cog} = \dfrac{\sum_i W_i \vec{R}_i}{\sum_i W_i}$ above any balance points below:

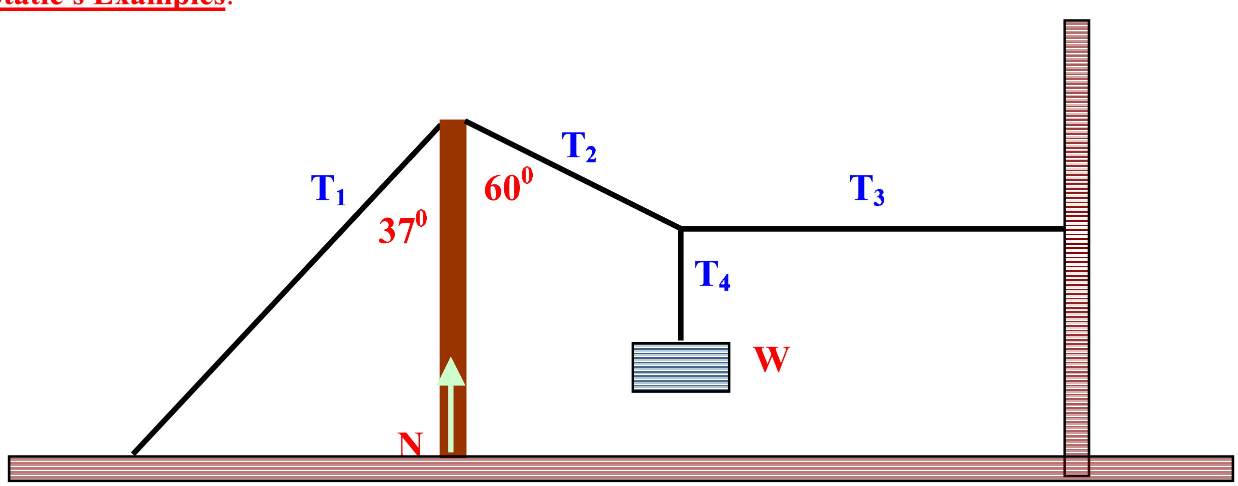
A body whose COG is above its base of support will be stable if a vertical line projected downward from the COG falls within that base of support.



The COG is the object COM if $\vec{g}$ is constant over the extent of the object.

Low COG and extended support bases are conducive to stability.

Static's Examples:



Find W, the tensions in wires 1 and 2 and the force with which the beam pushes downward on its base given the beam weighs 960N and T_3 is 840N

$$\sum F_x = 0 \qquad \sum F_y = 0$$

In X, at the knot: $T_3 - T_2 Cos(30^0) = 0$

In Y, at the knot: $T_2 Sin(30^0) - T_4 = 0$

$$T_2 Sin(30^0) - W = 0$$

$$T_2 = T_3 / Cos(30^0) = 840 / Cos(30^0)$$

$$W = T_3 * Tan(30^0) = 840 * Tan(30^0)$$

$$\sum \tau_{About\_base\_of\_beam} = 0$$

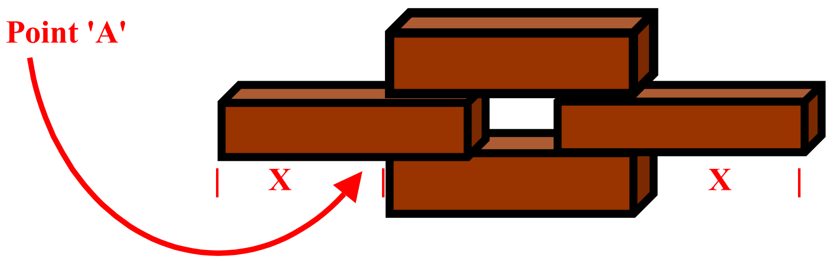
$$LT_1 Sin(143^0) - LT_2 Sin(120^0) = 0$$

$$T_1 = T_2 Sin(120^0) / Sin(143^0)$$

At the beam base $N - 960 - T_1 Cos(37^0) - T_2 Cos(60^0) = 0$

Solve for N.

For the four bricks below each of length L, find the maximum distance X that the interior bricks may shift outward before the structure tips.



The load of the top brick distributes evenly on the two middle bricks.

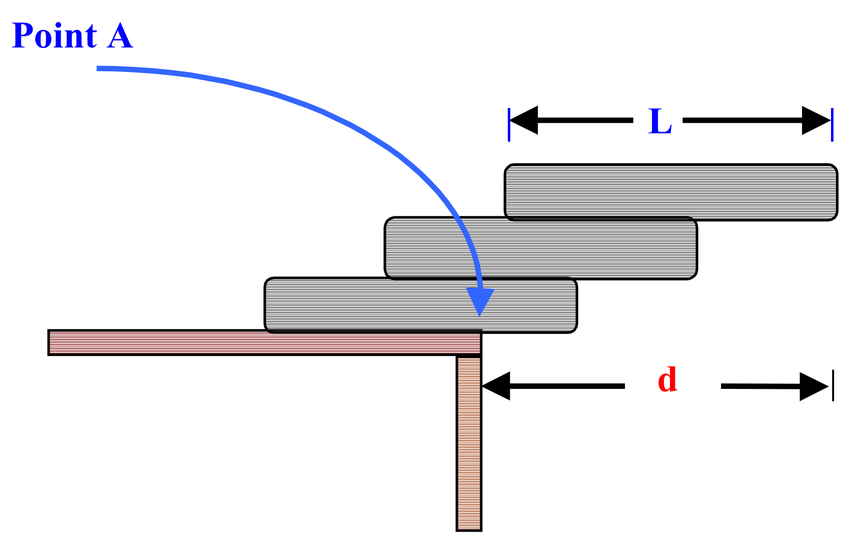
$$\sum \tau_{About\_A} = 0$$

From the top brick, $\tau = -\{L - X\} * \frac{W}{2}$

From the middle brick itself, $\tau = \{X - \frac{L}{2}\} * W$

These two equations give: $X = \frac{2}{3} * L$

Find the maximum overhang d such that the books are stable in the picture below:



Stability requires that a line drawn downward from the COG fall within the support base.

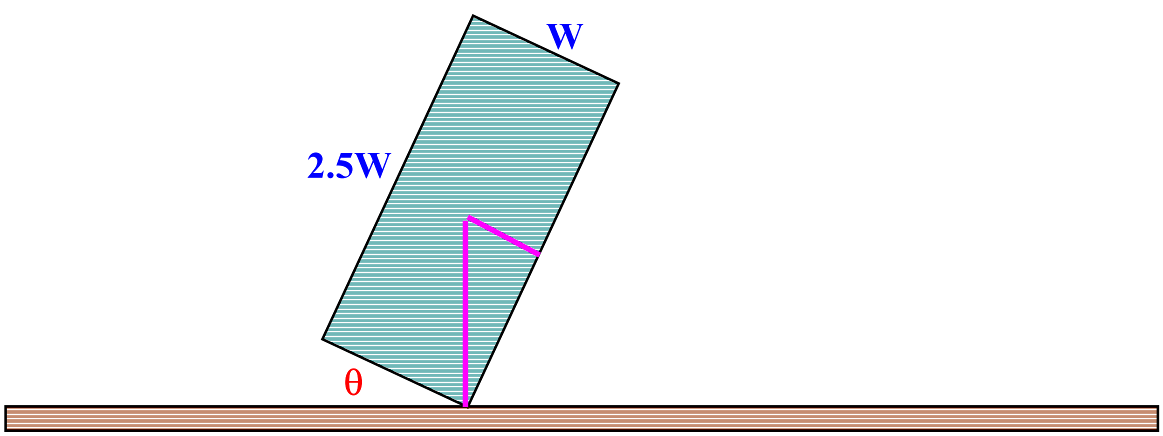
The top book must be no more than **L/2** beyond the book just beneath.

The COG for the combination of the top book and the next lowest is at **L/4** inward from the right edge of the middle book and that must be above the supporting edge of the bottom book. That is the second book is **L/4** beyond the bottom book.

Finally the combination of the two top books with the bottom one has a COG at **L/6** inward from the right edge of the bottom book. The overhang of the bottom book is therefore maximally **L/6**:

$$d = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} = \frac{11}{12}L$$

A uniform block is 2.5 times tall as it is wide. Find the angle 'theta' at which the block topples.



The COG for the block is at (W/2, 2.5W/2)

Rotating beyond $Tan^{-1}\frac{\frac{W}{2}}{\frac{2.5W}{2}} = Tan^{-1}0.4 = 21.8^{0}$ **topples the block**.

Fluids& Solids

Matter has four possible states:

1) **Solid**
2) **Liquid**
3) **Gas**
4) **Plasma**

Bulk properties of matter like density, volume, conductivities, etc. depend on atomic arrangements and interactions between atoms within the matter.

Gases generally have small atomic interactions, large volumes, low density, and atomic or molecular motions that may be treated independently for each atom or molecule.

Liquids and solids are localized due to large interatomic forces. A macroscopic treatment of the entire system on scales large in comparison to average atomic spacing is necessary in quantifying these systems.

Plasmas consist of free electrons and positively charged ions at high temperature. Plasmas are electrically neutral overall, as are the other states of matter, but exhibit vastly different electromagnetic characteristic in comparison to the other matter forms.

These four types of matter subdivide into two general categories:

1) **Solids**
2) **Fluids**

Solids have either **crystalline or amorphous** atomic structure, which determine in part the physical properties of the solid such as **reactivity, conductivity, malleability**, etc.

Crystalline solids have a repetitive orderly placement of individual atoms into a lattice. Amorphous materials have randomly arranged atoms with no definite structural subunit. Non-uniform overlapping of atomic layers changes the physics of amorphous solids.

Liquids, gases and plasmas are **fluids**. Inability to sustain **tangential shearing** and **continuous deformation of form** when subject to shearing stress define fluids.

Elasticity and Solids

Deformation of a solid subjected to **tensile, compressive, shearing or hydraulic** stress depends on the relative magnitude of these forces.

In a limited range **(elastic region)** of applied forces, the system responds linearly in proportion to the applied force.

The **stress** on the solid is the applied **force per unit area**.

The **strain** resulting from the above stress is the **fractional deformation** that results.

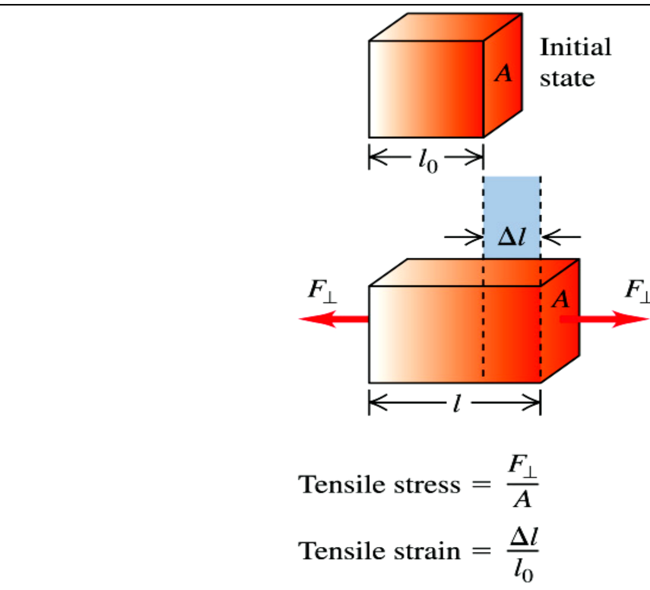
In the elastic region of a given material strain is linearly proportional to stress and materials return to their initial configuration when applied forces cease.

Beyond the **elastic limit,** an object enters into a **plastic region** where permanent deformation takes place and fractures when the material **ultimate strength** obtains.

Tensile Stress

For objects compressed or stretched along the length of the object, the **Young's Modulus** linearly relates tensile stress or compressive stress to tensile strain:

$$\frac{F_\perp}{A} = Y\frac{\Delta L}{l_0}$$



Units of the Young's Modulus are N/m^2

A N/m^2 is a **Pascal** Pa which is a unit of **pressure** or **force per unit area**.

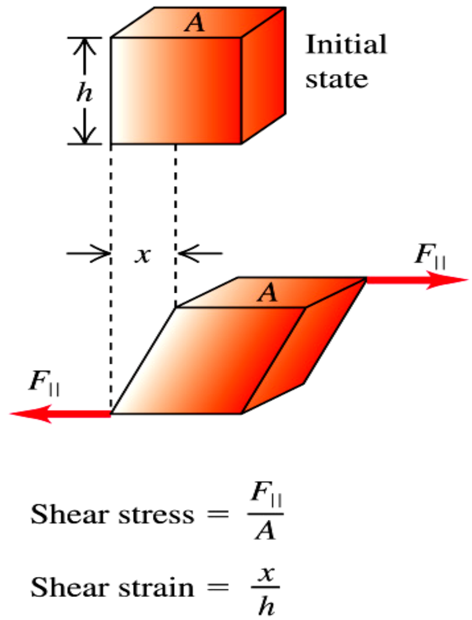
Shear Stress

If the object is being **sheared** at its top and bottom surfaces, then the constant of proportionality between stress and strain in the elastic region is the **Shear Modulus**:

$$\frac{F_{\parallel}}{A} = S\frac{x}{h}$$

For large S materials, shearing is more difficult.

For liquids, S → 0.



Hydraulic Stress

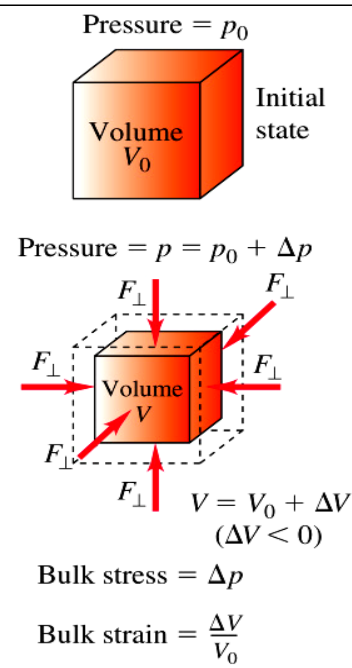
In bulk compressions/expansions, the **Bulk Modulus** relates pressure to volume changes:

$$\frac{F}{A} = -B\frac{\Delta V}{V_0}$$

$$\Delta p = -B\frac{\Delta V}{V_0}$$

Reciprocal of the Bulk Modulus is **compressibility**:

$$k = -\frac{\Delta V}{V_0} / \Delta p.$$



Fracture

Stress is linearly proportional to strain only up to strains of $< 1\%$

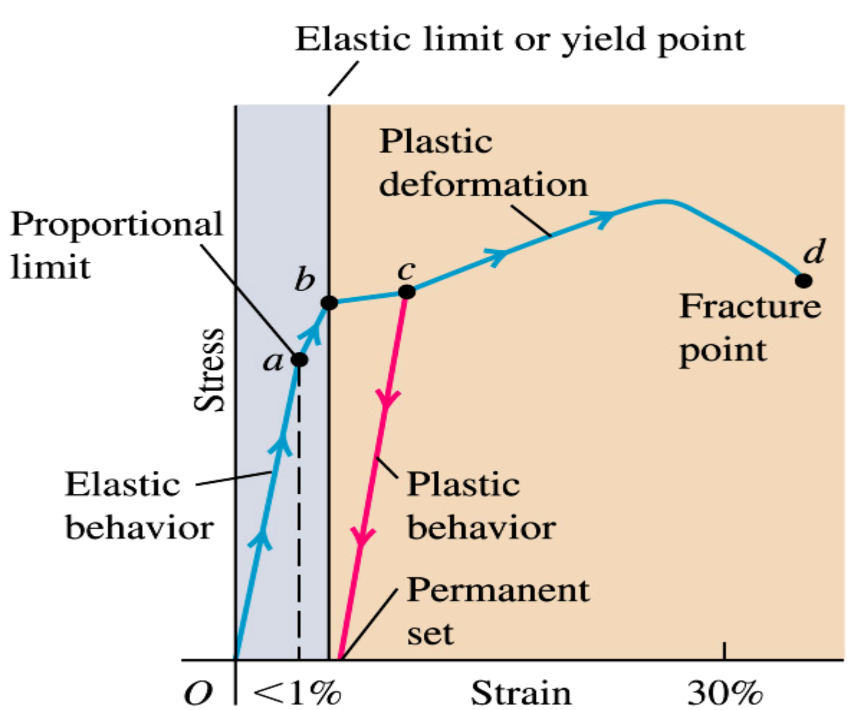
After a 1% deformation, materials pass the **elastic limit** and deformation is irreversible.

During **plastic flow** small increments of stress $\frac{F_{\perp}}{A}$ result in high material strain $\frac{\Delta L}{l_0}$.

Material **fracture** takes place for stress exceeding the **ultimate strength** of the material.

Materials fracturing just after the **yield point** are **brittle materials**.

Materials with extended plastic response are **ductile**.



Domes and Arches

A table of material ultimate strengths indicates that concrete **compressive** strength is an order of magnitude greater than its tensile strength.

The use of semicircular **arches** in architecture converts lintel tensile stress into compressive stress of an arch at the expense of having to buttress horizontal forces.

Circular **domes** are both buttressed and supported along their circumference by a tension ring to offset the large horizontal forces associated with this design.