Energy:

The **energy** of an object is the **scalar sum** of all contributing types or forms of energy that the object posses.

Energy has many **forms** and an object may posses some of any or all of these forms:

1) **Mechanical Energy**
2) **Thermal Energy**
3) **Electrical Energy**
4) **Chemical Energy**
5) **Nuclear Energy**
6) **…**

The utility of the concept of 'energy accounting' derives from the fact that in an isolated system the **total energy content is conserved**.

Accounting for all system energy in the initial state and then letting this state dynamically evolve into a final state where we again evaluate total energy, then the **conservation law** $E_f = E_0$ can be used to discern the physics of such a process.

The SI units of energy is the Joule: $1J = 1N \cdot m$

In CGS units energy is Ergs and in British units we have the $ft \cdot lb$**.**

Mechanical Energy:

A system has **mechanical energy** by virtue of its motion or velocity and also resulting from its physical configuration in space.

Energy associated with particle motion is **kinetic energy** and the energy of a system resulting from a particular spatial configuration is known as **potential energy**.

Work done on a system can change both the kinetic and potential energies.

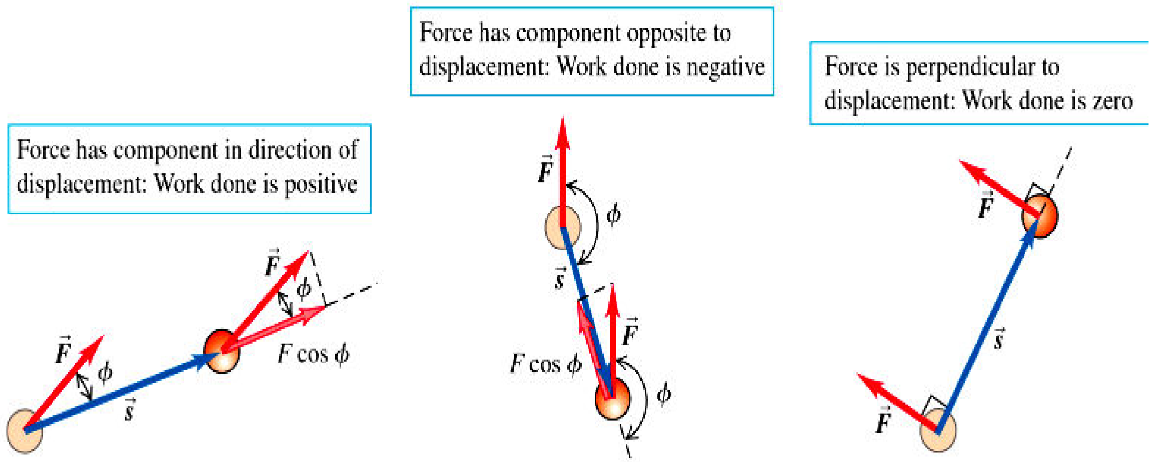
Work:

The physics definition of **work** gives a quantitative way to determine the effects **external forces** have on the systems energy content.

Example: Applying an external force in the direction of an objects motion will add to its velocity and therefore its kinetic energy content

Work is **force applied over a distance**. For **constant forces**, the vector product of the applied force and the resulting displacement is the work done: $W = \vec{F} \bullet \vec{d}$.

$$W = \vec{F} \bullet \vec{d} = |\vec{F}||\vec{d}|Cos(\phi)$$



$|\vec{F}|Cos(\phi)$ **is the component of** $\vec{F}$ **along the direction of displacement.**

When $\vec{F}$ **is perpendicular to** $\vec{d}$ **then this particular force does no work**.

Energy transferred **to a system** by application of an external force is **positive work.**

Energy extracted **from a system** by application of an external force is **negative work**.

If more than one force is present then we can either vector sum the forces and get W_{net} from this force **or** we can get individual work quantities from each force separately and scalar sum these to get $W_{net} = W_1 + W_2 + \ldots$

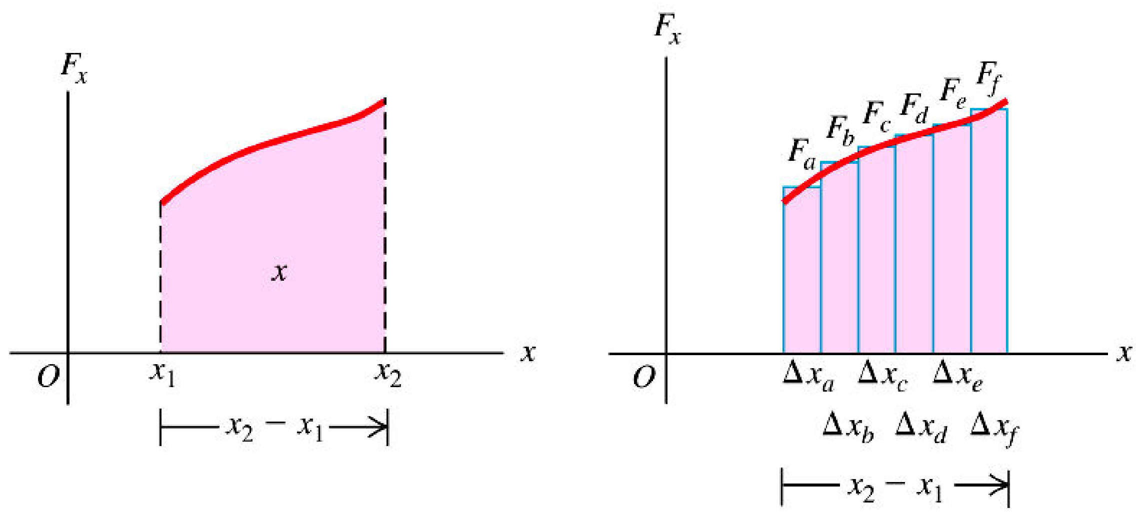
If the force is not a constant in magnitude or direction, then we must integrate to solve the problem as follows:

In 3-dimensions:

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{F} \bullet \overrightarrow{\Delta r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$$W = \lim_{\Delta x \to 0} \sum F_x \Delta x + \lim_{\Delta y \to 0} \sum F_y \Delta y + \lim_{\Delta z \to 0} \sum F_z \Delta z$$



Variable force example: Work done by a spring:

In the **elastic region**, a **restoring force** on a spring stretched or compressed beyond its **relaxed state** obeys **Hooke's Law**:

Restoring force is linear in the displacement away from equilibrium $F_x = -kx$

k is the **spring constant**, SI units of **N/m** and an indicator of spring stiffness.

To find work done by a spring stretched or compressed beyond equilibrium, write $W_s = \vec{F}_{AVG} \bullet \vec{d}$ using an average force since the spring force is variable.

$$W_s = \frac{F_0 + F_x}{2}(\pm\hat{i}) \bullet x(\mp\hat{i}) = -\frac{1}{2}kx^2$$

The spring system tends to a **minimal potential energy configuration** with zero stretching and/or compression.

Note if an external agent is to supply the force needed to compress or stretch the spring, then this agent must do work in the amount $W_{agent} = +\frac{1}{2}kx^2$.

Constant force example: Work Done by Gravity:

Moving an object from ground upward, we have $\vec{F_g} = -mg\hat{j}$ and $\vec{d} = +h\hat{j}$

The work done by gravity is $W_g = \vec{F_g} \bullet \vec{d} = -mgh\hat{j} \bullet \hat{j} = -mgh$

Gravity does negative work, the system tending toward **minimal potential energy**.

A person lifting the object will have to minimally overcome the weight force mg and supply a force $\vec{F_p} = +mg\hat{j}$. The person does work: $W_g = +mgh$

Kinetic Energy:

Particle kinetic energy is: $K = \frac{1}{2} m V^2$

The **Work-Energy Theorem** relates changes in the objects kinetic energy to work done by all external forces.

Consider the $\phi = 0$ case for applied force is in the direction of the displacement:

$$W = \vec{F} \bullet \vec{d} = |\vec{F}||\vec{d}| Cos(\phi) = F \cdot d = ma \cdot \Delta x$$

Kinematics equation: $\Delta x = \dfrac{V_f^{\,2} - V_o^{\,2}}{2a}$

$$W = ma \cdot \Delta x = \tfrac{1}{2} m V_f^{\,2} - \tfrac{1}{2} m V_0^{\,2} = \Delta KE$$

The work done by all forces is equal to the change in the object kinetic energy.

$$W > 0 \Rightarrow \Delta KE > 0 \Rightarrow V_f > V_0$$

$$W < 0 \Rightarrow \Delta KE < 0 \Rightarrow V_f < V_0$$

Potential Energy

Configuring a physical system with masses displaced relative to a ground or with springs compressed /stretched relative to a relaxed length means that configured in the system is potential for doing work.

Gravitational potential energy depends on the configuration of mass within the system. **Elastic potential energy** depends on extension /compression away from equilibrium length of any springs in the system. Both of these and other forms of PE may be present.

Path independence: Work done during a system configuration change only depends on the starting and ending configurations and not on any intermediate states.

The path over which a system evolves from its starting configuration into its finishing configuration is irrelevant. Only the start and finish configurations are relevant.

If this is not the case, then we cannot quantify a system using potential energy terms.

Path independence holds true for **conservative forces** like gravity or spring forces, but not for forces such as friction or directionally varying forces like tensions.

A unique feature of conservative forces is that its vector direction is fixed regardless of the direction in which an object subjected to this force moves. In contrast, non-conservative force vectors continually change in direction during the object motion.

For conservative forces, $\Delta PE = -W$

The change in system potential energy is equal to the negative of the work done on the system by the **conservative forces** associated with these potentials.

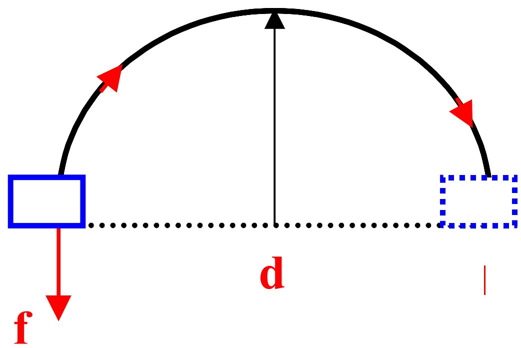
Friction ⇔ **Non-conservative force.**

Consider the amount of work done by the friction force on the object over the course of the two paths shown here: Take the coefficient of friction of the page to be μ_k .

Path 1:

$$W_f = -\mu_k N d$$

Path 2:

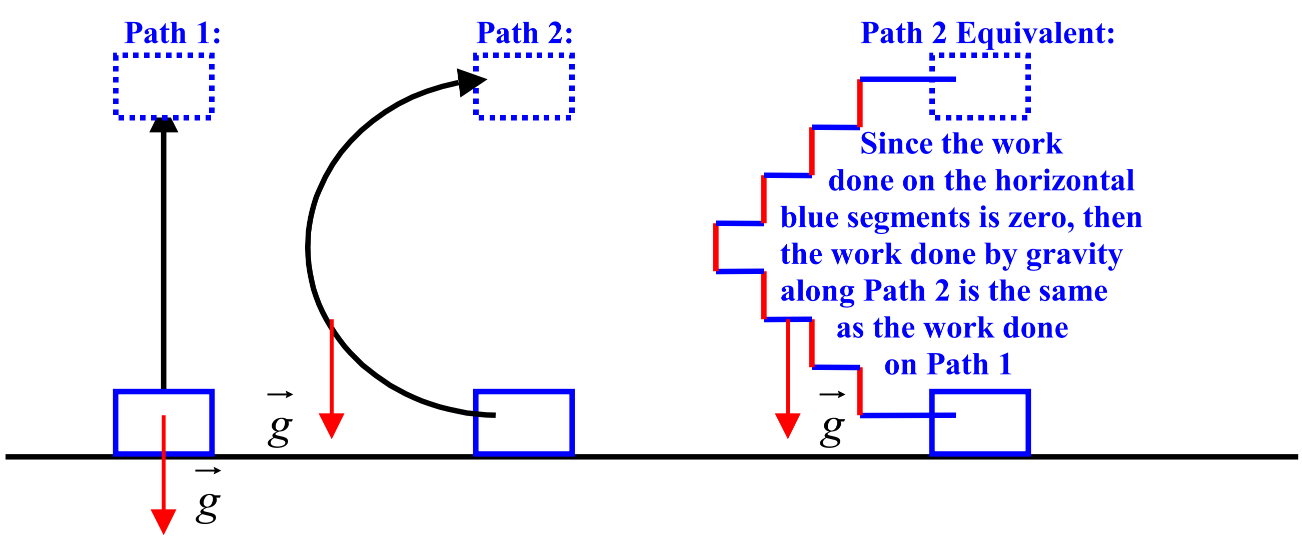


$$W_f = -\mu_k N \frac{\pi}{2} d$$

The work done by the non-conservative friction force **depends on the path** taken.

Gravity ⇔ Conservative force,

Recall the direction of $\vec{g}$ always points towards earth's center.



The benefit we gain with **conservative forces** is that it becomes possible to derive analytic expressions for the potential energy terms associated with these forces

For Gravitational Potential Energy

Using $\Delta PE = -W$ we have:

$$\Delta PE = -W_g = -\left[-mg(H_2 - H_1)\right]$$

$$\Delta PE = mg(H_2 - H_1)$$

At height H **above what you set as the potential ground**, $PE = mgH$.

For Elastic Potential Energy:

$$\Delta PE = -W_s = -\left[-\frac{1}{2}kx_f^{\,2} - \left(-\frac{1}{2}kx_i^{\,2}\right)\right]$$

$$\Delta PE = \left[\frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}\right]$$

The spring potential energy associated with any elongation or compression, x_f **beyond the relaxed state** is $\left[\frac{1}{2}kx_f^{\,2}\right]$.

Restatement of the Work-Energy Theorem

The Work Energy Theorem is: $W_{All\_Forces} = \Delta KE$

The LHS separates into two terms corresponding to work done by conservative forces and work done by non-conservative forces.

Using the result $W_{Conservative\_Forces} = -\Delta PE$

$$W_{Non\_Conservative\_Forces} - \Delta PE = \Delta KE$$

$$W_{Non\_Conservative\_Forces} = \Delta KE + \Delta PE$$

Conservation of Total Mechanical Energy

In the absence of non-conservative forces within a mechanical system, the total mechanical energy $E_{Mech} = KE + PE$ is a **constant.**

The work energy theorem gives the result:

$$0 = \Delta KE + \Delta PE$$

$$\Delta E_{mech} = \Delta KE + \Delta PE = 0$$ Systems without losses.

<u>In an isolated system where only conservative forces are involved, the mechanical energy of the system is constant.</u>

Conservation of Total Energy:

If conservative forces only are acting, then $0 = \Delta KE + \Delta PE$.

When forces such as friction are present, include these forces and conserve **total energy** by appropriately defining the 'isolated system' to include bodies into which dissipative heat flows as a result of the friction between this body and the object under study:

For Friction we have seen $W_{nc} = \overrightarrow{f_k} \bullet \vec{d} = -\mu_k Nd$

This work detracts from the objects overall energy and transfers this detracted amount into heat at the surface in contact with the object.

Writing the work energy theorem,

$$-\mu_k Nd = \Delta KE + \Delta PE$$

$$0 = \Delta KE + \Delta PE + \mu_k Nd$$

$$0 = \Delta E_{mech} + \Delta E_{thermal}$$

Which is applicable now to the redefined isolated system.

Internal Energy

It may also be the case that some internal mechanism or process internally can generate usable energy that converts into mechanical and/or thermal energies.

The easiest example here is that of a person running up a hill thereby utilizing internal muscle energy to acquire kinetic, potential, and thermal energies.

The conservation of energy for the isolated system becomes:

$$0 = \Delta E_{mech} + \Delta E_{therm} + \Delta E_{\text{int}}$$

If there are **external forces** that are not part of our isolated system and which are applied to the isolated system as a whole, then the work energy theorem is:

$$\overrightarrow{F_{ext}} \bullet \vec{d} = \Delta E_{mech} + \Delta E_{th} + \Delta E_{\text{int}}$$

Or,

$$W_{F_{ext}} = \Delta E_{Total}$$

Power:

Power is the work done on an object or by an object per unit time.

Power is the time rate of energy transfer.

$$P_{AVG} = \frac{W}{\Delta t}$$ Is the **average power** delivered.

$$P_{AVG} = \frac{W}{\Delta t} = \frac{F\Delta r}{\Delta t} = Fv_{AVG}$$

At any instance in time, using the instantaneous velocity:

$$P_{inst.} = \frac{dW}{dt} = \frac{d}{dt}(\vec{F} \bullet \vec{d}) = \vec{F} \bullet \vec{v}$$

The SI units of power are $1 J/s = 1 \cdot Watt = 1W$

Other units include hp, $ft \cdot lb/s$ **and** $BTU/hour$.

One common household measure is the **kilowatt-hour** which is a billable amount of energy the electric company supplies corresponding to an energy in Joules of:

$$1kW \cdot hour \times \frac{3600s}{hour} \times \frac{1000W}{1kW} = 3.6MJ$$