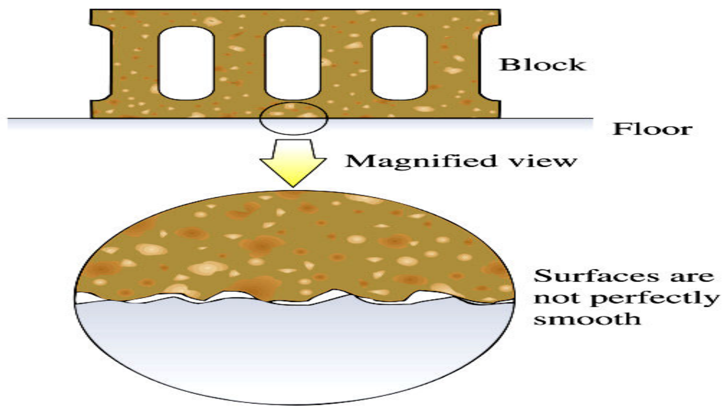
Friction:

Experimentally the following features are observed to be true of the force of friction:

1) Friction always **opposes the motion**. The force is **dissipative** and its direction is parallel to the surface of the object in motion.

1) The magnitude of the friction force is **proportional to the objects normal force** $f_s \le \mu_s N$ for static friction and $f_k = \mu_k N$ for kinetic friction. **μ_s and μ_k** are the **coefficients** of static friction and kinetic friction respectively.

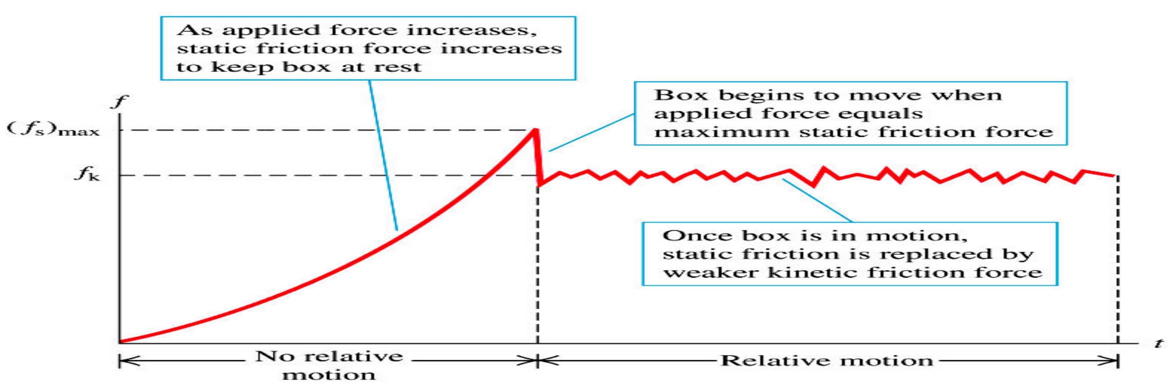
3) **$\mu_s > \mu_k$.** This makes sense since a stationary object forms stronger **contact welds**. The kinetic force of friction is less than the static force of friction.



4) f_s and f_k are **independent of the surface area** of contact and object velocity.

5) When $\vec{F}$ *Parralel* to the surface exceeds $f_{s,\max} = \mu_s N$ the object breaks free.

6) The magnitude of static friction force is equal to the magnitude of $\vec{F}$ *Parallel* that is applied up until the object begins to slide.



Drag Force and Terminal Speed:

For objects moving through a **fluid** such as the atmosphere, a friction **drag force** $\vec{D}$ results at the fluid-surface interface.

For low particle velocities when flow is **laminar or streamline**, the drag force on a spherical object of radius r follows **Stokes' Law**: $D = 6\pi\eta r v$

Here η is the fluid **viscosity**, a measure of internal friction in the fluid layers.

For **Reynolds number** $N_R = {2\rho v r}/{\eta} \geq 10$ fluid flowing across a spherical surface begins to flow **turbulently** and drag force depends on the square of the velocity:

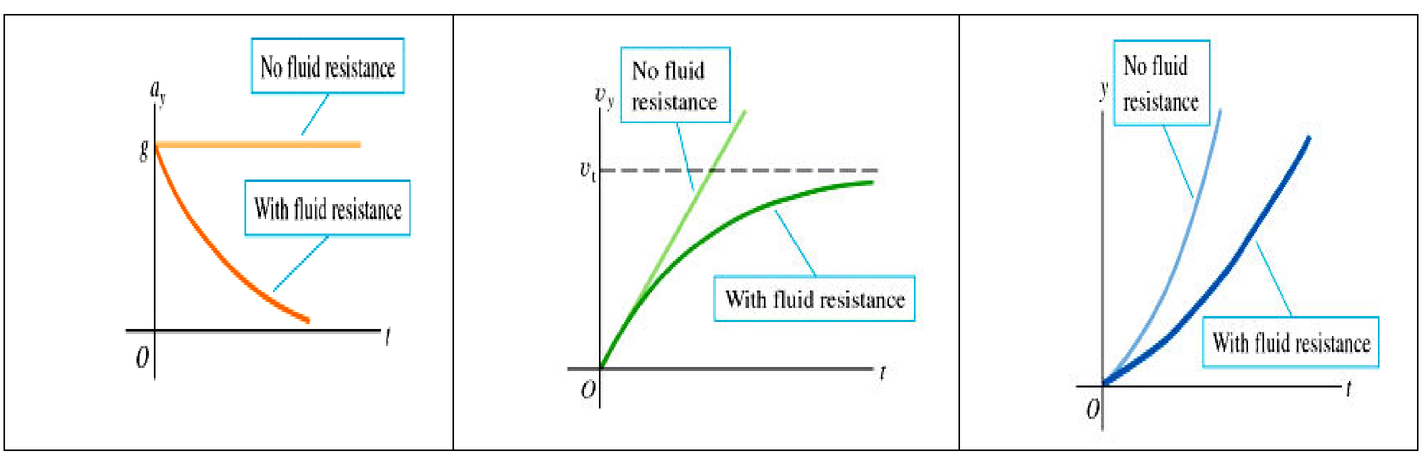
$$D = \frac{1}{2} C \rho A v^2$$

ρ is fluid density, A cross-sectional area, v velocity and C a drag coefficient.

By setting $D = mg$ and solving for free fall **terminal velocity**:

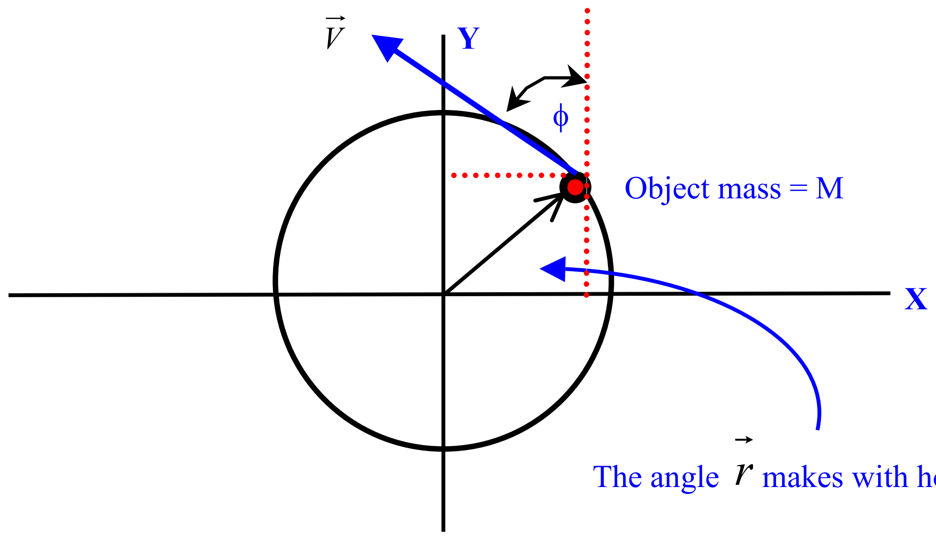
$v_t = \sqrt{\dfrac{2mg}{C\rho A}} = const. \Rightarrow$ The object free fall acceleration has ceased.

Sky Diver ➔125 mph **Baseball ➔94 mph**
Ping-Pong Ball ➔20 mph **Parachutist ➔11 mph**



Uniform Circular Motion:

Given an object in a fixed circular path motion and **constant velocity vector magnitude**, there exists acceleration since the **velocity vector direction** is changing continuously during this motion:



The **period** of revolution is the circumference divided by the velocity magnitude:

$$T = \frac{2\pi * r}{v}$$

The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ with $x = rCos(\phi)$ and $y = rSin(\phi)$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = -vSin(\phi)\hat{i} + vCos(\phi)\hat{j}$$

$$\vec{v} = -v\frac{y}{r}\hat{i} + v\frac{x}{r}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = -\frac{v}{r}v_y\hat{i} + \frac{v}{r}v_x\hat{j}$$

$$\vec{a} = -\frac{v}{r} v Cos(\phi)\hat{i} - \frac{v}{r} v Sin(\phi)\hat{j}$$

$$|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2} = \frac{v^2}{r}$$

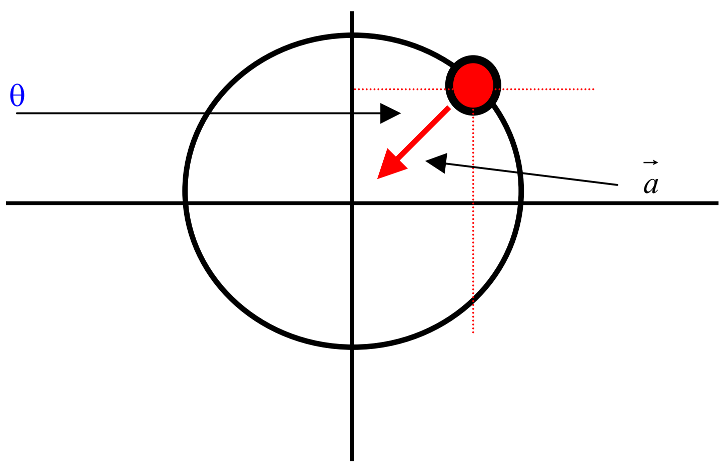
$$a_{rad} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

The direction of $\vec{a}$ is:

$$Tan(\theta) = \frac{a_y}{a_x} = \frac{-\frac{v^2 Sin(\phi)}{r}}{-\frac{v^2 Cos(\phi)}{r}} = Tan(\phi)$$

Angle θ = Angle ϕ i.e, acceleration is centripetal or center seeking.

The angle $\vec{a}$ makes with horizontal is θ



For **non-uniform circular motion**, there is both a centripetal acceleration due to the changing direction of the velocity vector and a **tangential acceleration** due to the changing velocity vector magnitude.

$$a_{rad} = \frac{v^2}{r} \qquad a_{\tan} = \frac{d|\vec{v}|}{dt}$$

Summary of Uniform Circular Motion:

Particles executing uniform circular motion (constant $|\vec{v}|$ magnitude) have

centripetal acceleration $a = \frac{V^2}{r}$. According to Newton's 2nd Law there will be a

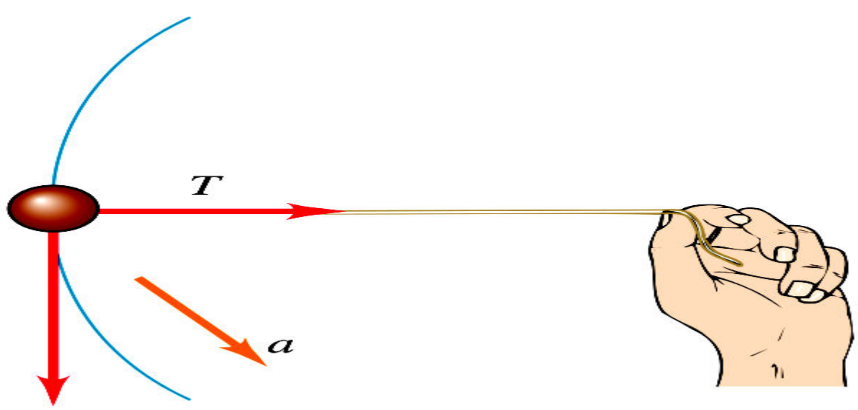
centripetal force $F = m * \frac{V^2}{r}$ that is directed towards the center of the circle.

In solving problems which have an object constrained to move in uniform circular motion, we can equate the component of the constraining force keeping the object moving along its circular path to the centripetal force.

Non-uniform Circular Motion:

For non-uniform circular motion ($|\vec{v}|$ magnitude is no longer constant), there is **centripetal acceleration** due to the changing direction of the velocity vector and **tangential acceleration** due to the changing velocity vector magnitude.

$$a_{rad} = \frac{v^2}{r} \qquad a_{\tan} = \frac{d|\vec{v}|}{dt}$$



$$|\vec{a}| = \sqrt{a_{rad}^{\;2} + a_{\tan}^{\;2}}$$

Gravitation and Kepler's Laws

In addition to three laws of motion, Isaac Newton also discovered a **law of gravitation** that went unchanged for roughly 250 years and required Einstein for its revision.

Gravity is the **weakest among the four fundamental forces**.

Gravity is responsible for processes ranging from particle-particle attractions, to galactic scale events like the formation of galaxy clusters and superclusters.

Newtonian gravity was the leading theory of gravity until the early 1900's when the predictions of the **General Theory of Relativity**, subsequently verified, dramatically changed the view of gravity as well as future approaches to understanding the nature of the other known forces.

Newton's Law of Gravitation

Two masses M_1 and M_2 separated by a distance r are attracted according to:

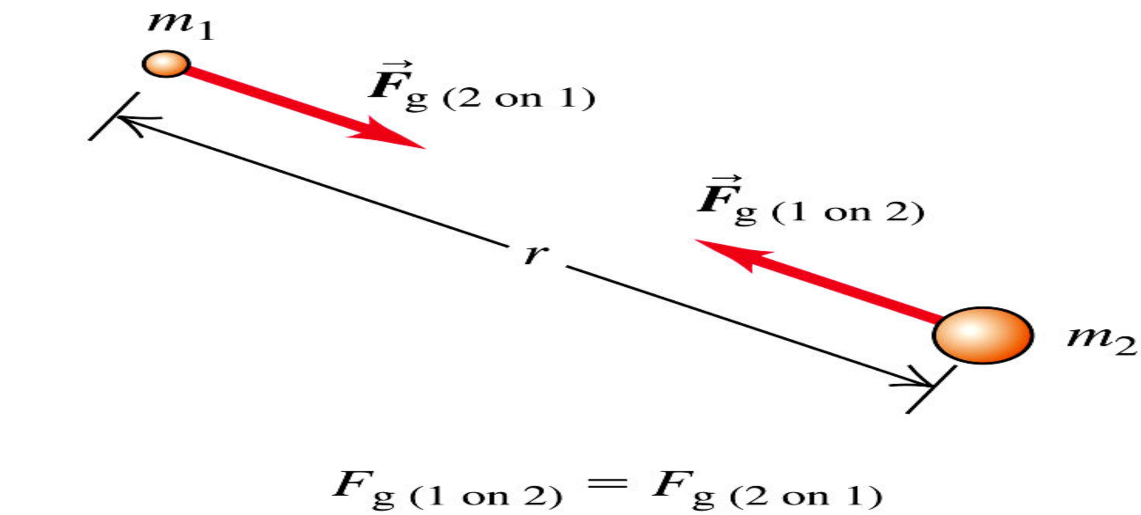
$$\vec{F} = G\frac{M_1 M_2}{r^2}\hat{r}$$

Along a line joining the centers of M_1 and M_2.

Note the following:

1) **Proportional to the product $M_1 * M_2$**
2) **Inversely proportional to the square of r**
3) **Direction is attractive for both M_1 and M_2**
4) **Newton's 3rd law ➔** $\vec{F}_{12} = -\vec{F}_{21}$

G is Newton's Gravitational Constant: G = 6.67x10^{-11} Nm^2/kg^2



Notice the **action at a distance** problem associated with this model.

Looking at an earth-apple system, the question of how **# 4** above results in our seeing the apple falling to earth and not the earth accelerating up to the apple is answered:

$$F_{Earth} = -F_{Apple} = -M_{Apple}\,g$$

$$a_{Earth} = \frac{F_{Earth}}{M_{Earth}} = \frac{-M_{Apple}\,g}{M_{Earth}}$$

The apple's acceleration is g , but earth's acceleration is only a small fraction of g

Also, an object of mass M_1 on the surface of earth experiences the gravitational pull of the earth as $M_1 g$. Equating to Newton's Gravitational Law: $G\frac{M_1 M_2}{r^2} = M_1 g$

Gives g dependent only on the parameters that characterize earth and G

$$G\frac{M_{Earth}}{r_{Earth}^{\;2}} = g$$

Shell Theorems

A uniform spherical shell of matter attracts a particle that is outside as if all the shells mass were concentrated at its center.

We have assumed this is true of the earth, and this is close to the actual reality:

1) g Is not a constant over the surface of the earth
2) ρ Of earth in non-uniform with depth (crust, mantle, outer core, inner core)
3) Earth is not spherical: $R_{pole} < R_{equator}$
4) Earth's angular rotation ω makes an object lighter at the equator:

At the poles, $N - mg = 0$ such that N (which is the weight) is equal to mg :

At the equator, centripetal forces produce a sum of forces that is not identically zero:

$$N - mg = -\frac{mv^2}{R_{equator}}$$

$$N = m\{g - \frac{v^2}{R_{equator}}\}$$

The term in parentheses is an **effective** value of g at the equator. The value is about a **third of a percent less than** the $g = 9.80$ earth average.

A uniform shell of matter exerts no net gravitational force on a particle located inside the shell.

For uniformly dense objects, then burrowing downward ➔ two opposing effects on the gravitational force:

1) **Decreasing R ➔ Increasing F**
2) **Moving into a shell ➔ Decreasing F**

For uniformly dense objects, factor two is more pronounced and the object moves uniformly to a region of zero gravity.

For the earth no such uniformity exists and what is actually found is:

1) Factor one is initially larger than factor two so g increases (light crust/mantle)
2) Factor 2 eventually wins out and gravity diminishes upon further descent.

Systems of Masses

For systems of many masses, **superposition** means we can use Newton's Law of Gravity to determine the forces present on individual masses by proceeding with vector arithmetic

$$\vec{F}_{m1} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n}$$

In two dimensions:

$$F_{m1_x} = F_{12_x} + F_{13_x} + F_{14_x} + \ldots + F_{1n_x}$$

And

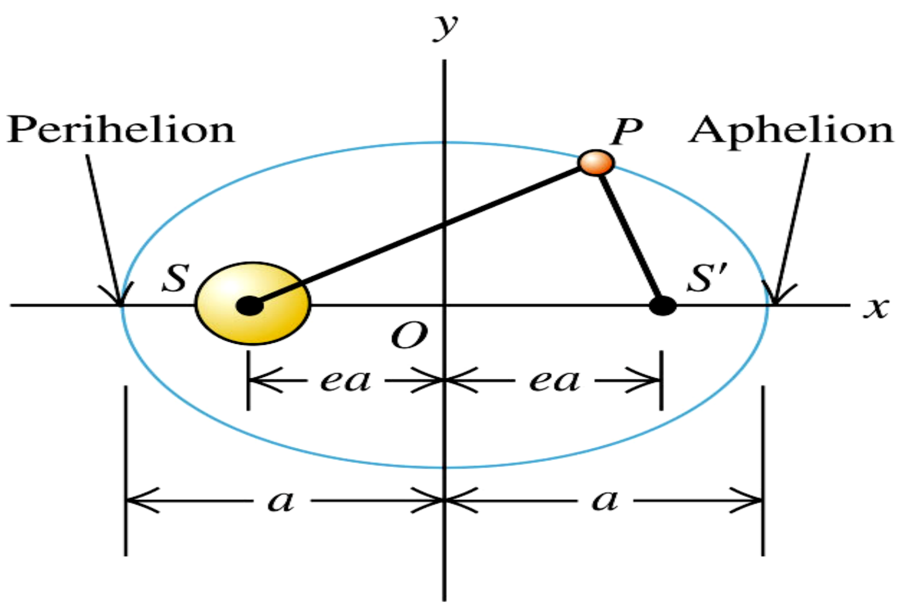
$$F_{m1_y} = F_{12_y} + F_{13_y} + F_{14_y} + \ldots + F_{1n_y}$$

$$\left|\vec{F}\right| = \sqrt{F_{m1_x}{}^2 + F_{m1_y}{}^2}$$

Kepler's Laws

Johannes Kepler using planetary data collected by Tycho Brahe deduced the following **three laws of planetary motion**:

First Law: Path of each planet about the sun is an ellipse with the sun at one focus:

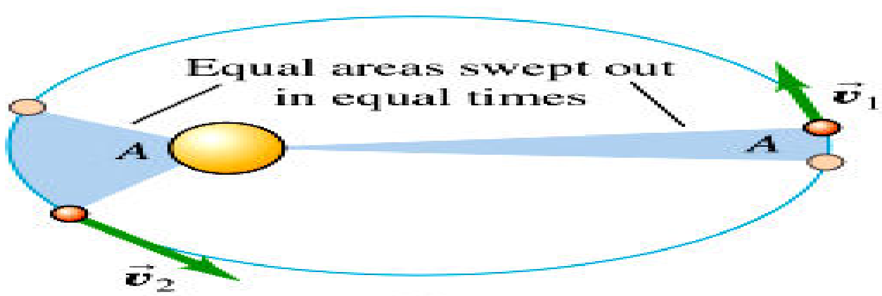


The ellipse has SP+S'P = Constant.

The horizontal extent of the ellipse is its major axis. Half of this is a semi-major axis.

The vertical dimension is the minor axis. Half of this is the semi-minor axis.

Second Law: Planets move so an imaginary line drawn from the sun to the planet sweeps out equal areas in equal time intervals.



More kinetic energy at perihelion ➔ faster orbital speed. Newton found that this law derives from conservation of angular momentum. A sector has area $\frac{1}{2}r^2\theta$. Constant $\frac{\Delta A}{\Delta t}$ ➔ constant $\frac{1}{2}r^2\frac{\Delta\theta}{\Delta t} \propto r^2\omega$ where '**omega**' the angular velocity is just the angular momentum $L = mr^2\omega$ divided by the mass.

Third law: $T^2 \propto a^3$ Here T is orbital period and a the semi-major axis of the orbit.

For objects in orbit constrained to a circular path by gravity, we equate centripetal force to gravitational force:

$$m\frac{V^2}{a} = G\frac{Mm}{a^2} \qquad \frac{V^2}{a} = G\frac{M}{a^2} \qquad v = 2\pi a/T$$

$$\frac{4\pi^2 a^2}{T^2 a} = \frac{GM}{a^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) * a^3$$

Note that if T is in units of **years** and a in units of **AU**, this law simplifies:

$$T^2 = a^3$$

E.g., For Mars, a = 1.52 AU ➔ T = 1.88 years.

Geosynchronous Satellites:

Satellites that remain in orbit above a fixed point on the earth surface have several important applications in communications, weather, navigation, military…etc.

Finding the orbital distance above the earth of such a satellite requires only that we set its orbital period equal to **1 earth day = 86,400 s**:

From the 3rd law of Kepler $a = \left(\frac{T^2 GM}{4\pi^2} \right)^{\frac{1}{3}}$

$$a = \left(\frac{86{,}400^2 * 6.67x10^{-11} * 5.97x10^{24}}{4\pi^2} \right)^{\frac{1}{3}} \cong 42{,}200km$$

H = height above earth ~ 35,800 Km or about six Earth radii.