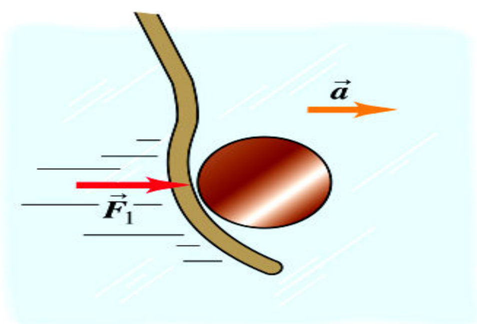
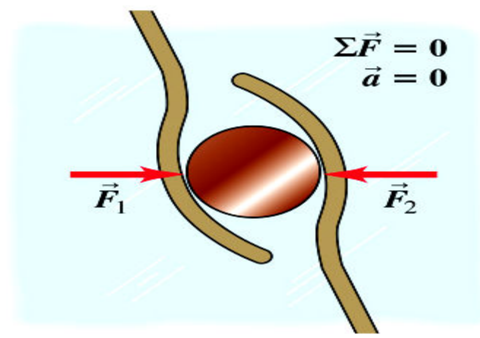
Dynamics, Force, and Newton's Laws:

When determining what causes motion or changes in motion we must examine the **forces** present that act on the particle whose dynamics we have an interest in understanding.

Of principal importance is the **net external force** acting on our object. The vector sum of all forces applied to the particle.

Jumping ahead slightly it turns out that if there exist a net external force on a particle, then it will accelerate. Zero net external force, implies particle acceleration will be zero.





Example: if there are three forces acting the particle and we know the components of each force $[F_{1x}, F_{1y}, F_{1z}]$ $[F_{2x}, F_{2y}, F_{2z}]$ $[F_{3x}, F_{3y}, F_{3z}]$, then the **net force** in **'x'** is $[F_{1x} + F_{2x} + F_{3x}]$ in **'y'** the **net force** is $[F_{1y} + F_{2y} + F_{3y}]$, and in **'z'** the **net force** is $[F_{1z} + F_{2z} + F_{3z}]$. This is called **superposition of forces.**

If these net force sums are not equal to zero, then there will be an acceleration of the particle that is proportional to the applied net force:

$$a_x = \frac{1}{m}[F_{1x} + F_{2x} + F_{3x}] \qquad a_y = \frac{1}{m}[F_{1y} + F_{2y} + F_{3y}]$$

$$a_z = \frac{1}{m}[F_{1z} + F_{2z} + F_{3z}]$$

a_x, a_y, a_z Are components of the acceleration vector and m the particle mass. The point here is that for a non-zero **net external force** the object will accelerate.

Quantitative treatment of the mechanics of classical, non-relativistic objects was set forth by **Isaac Newton [1642-1727]** in 1687 **Philosophiae Naturalis Principia Mathematica**

Newton's work was revolutionary in the impact that it had in turning the previous amalgam of superstition and empiricism into a scientific method relying on experimental data and then going beyond to provide a theory of phenomena founded in mathematics.

The statement that an object experiences acceleration proportional to the net external force acting on the object and that the constant of proportionality is the objects mass is **Newton's Second Law**.

The **2nd** law is best understood when Newton's **1st** and **3rd** laws are established. The former is a special limiting case of the Second Law and the later reminds us to **isolate** the object whose dynamics we wish to describe by the application of Newton's Second Law.

The **1st** law also called the **law of inertia** was discovered by Galileo (1564-1642). If no forces act on an object, then its velocity cannot change; i.e., no acceleration.

Rephrased, the **1st** law is: In the absence of external forces an object at rest or in motion with constant uniform velocity stays at rest or in motion with the same constant velocity.

From the **1st** law a definition of the **inertial reference frame**: if the frame of reference in which we make our experimental measurements is accelerating, results inconsistent with Newton's laws obtain. A reference frame that is at rest or moving with constant velocity such that Newton's laws hold true is an inertial reference frame {Non-accelerating}.

The **1st** law corresponds to the case $F_x = F_y = F_z = 0 \Rightarrow a_x = a_y = a_z = 0$ implying non-changing velocities.

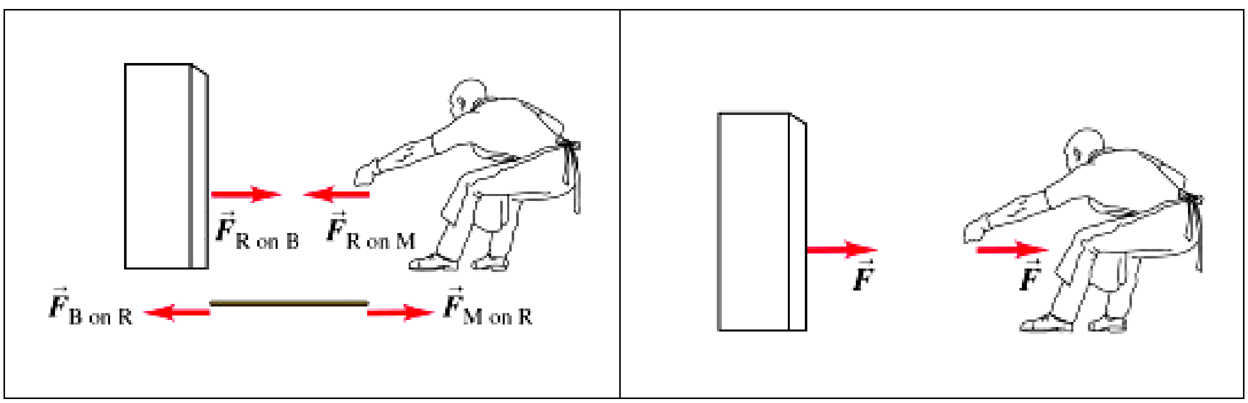
The **3rd** law is the **action-reaction law** stating: for every action or externally applied force on an object there is an equal in magnitude and oppositely directed reaction force from the object on the external agent applying that force.

When two bodies interact, the forces are equal in magnitude and opposing in direction.

The **3rd** law helps us understand the **2nd** law (not yet stated) by directing attention to a critical step in the steps taken in applying Newton's **2nd** law to solve mechanics problems.

For example, it might seem that if you were to push on a box, and the box pushes back on you with an equal and oppositely directed force that since the vector sum of these forces is equal to zero then this box, initially at rest, will stay at rest in accordance with the first law. We know that this reasoning must be incorrect since we can always relocate boxes small enough in mass by sliding them along the floor.

To correctly analyze the box dynamics we have to consider all the forces acting on the **box alone**. That is, **isolate** the box, list all the forces on the box, vector sum all these forces and then the **2nd** law tells us that $\vec{a}_B = \sum \vec{F_B} / m_B$. Note this says nothing about the forces acting on the person who is pushing the box. To know about the person's dynamics, we sum all the external forces acting on the **person only**: $\vec{a}_P = \sum \vec{F_P} / m_P$



The **2nd** law tells us how to relate an objects acceleration to applied net external forces. Being a **vector equation** it is applied independently in each of the coordinate directions.

$$F_{x,net} = ma_x$$

$$F_{y,net} = ma_y$$

$$F_{z,net} = ma_z$$

Associated with force is a new unit of measure called the Newton in SI units:

In SI units $1N = 1kg * ms^{-2}$

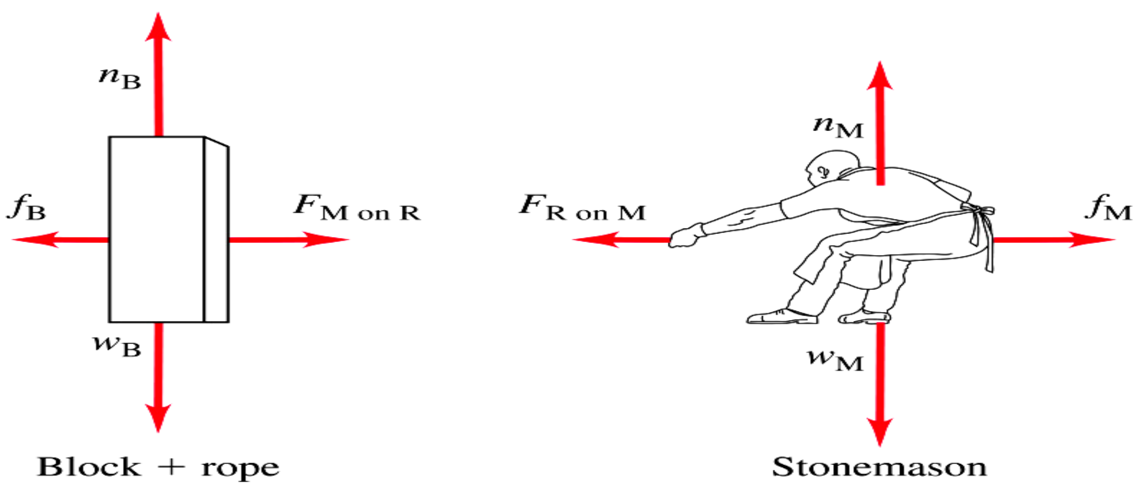
In CGS the unit of force is called the Dyne where $1dyn = 1g * cms^{-2}$

In British units a force is measured in Pounds where $1lb = 1slug * fts^{-2}$

A distinction here between mass and weight is appropriate. **Mass** is an intrinsic property of the object characterizing its inertia. **Weight** is the force the mass experiences due to any gravitational interaction. The measure of weight most familiar from our everyday lives is the pound, the British unit of force. On the earth, particle mass and weight are related by Newton's **2nd** law: $weight = mass * g$ with $g = 9.80ms^{-2}$

Application of Newton's Second Law

To use the **2nd** law, draw all the external forces to form a **free body diagram**.



The x, y, z components for each of the forces are resolved in a set coordinate frame and then component summed. Using $F_{x,net} = ma_x$, $F_{y,net} = ma_y$, and $F_{z,net} = ma_z$, the components of the acceleration vector are found.

The resulting magnitude of the acceleration vector is $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

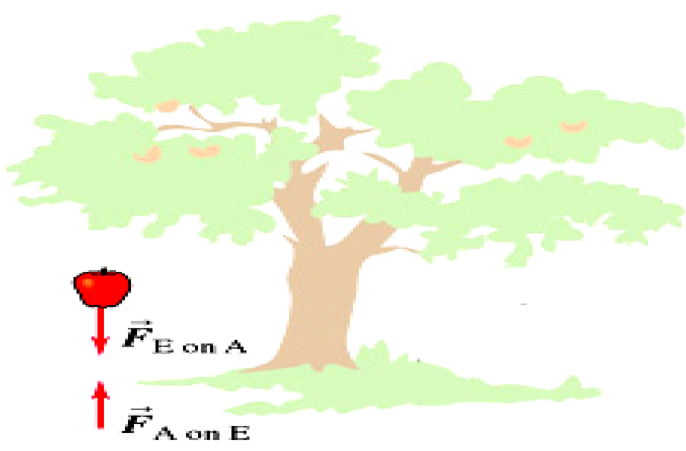
The direction of $\vec{a}$ may be determined with trigonometry.

Forces in Classical Mechanics:
Four forces typically encountered in Newtonian mechanics problems are:

1) **Gravitational Forces**
2) **Normal Forces**
3) **Tension Forces**
4) **Friction Forces**

1) The **gravitational force** on an object located on the earth is an **attractive** force that results from the earth's mass. This force is directed toward the center of the earth. If we imagine all the earth mass as being concentrated at the center point of the earth, then the force vector on an object points directly toward this mass concentration. The magnitude of the gravitational force is what we call weight and is equal to the product mg where $g = 9.80ms^{-2}$

An object's **apparent weight** will change if the reference frame in which weight is measured is accelerating or non-inertial.



2) **Normal forces** are Newton's 3rd law reaction forces on an object that results from the **contact** between the object and another surface. If not for the ground contact force 'pushing up' on a person standing on the surface of the earth, she or he would be in free fall toward the earth accelerating at $g = 9.80ms^{-2}$.

Normal force has the name because its direction is **perpendicular** to the surface with which it is in contact.

3) **Tension forces** arise in strings, cords etc., when these are used to transfer forces or make interconnects between objects in a problem. Ideally, strings and cords are massless and inextensible such that a force applied at one end is transmitted undiminished to the other end keeping the tension uniform throughout.

4) **Friction** is considered for the case where an object, being either at rest or in motion, is in contact with a second surface and forms **contact welds**, and the case where the object is moving through a **viscous fluid**.

Friction:
From an atomic perspective when two surfaces are in contact with each other, either at rest or in motion sliding across one another, then via electromagnetic interactions there forms **contact bonds** or **contact welds** that temporarily bond the surfaces together.

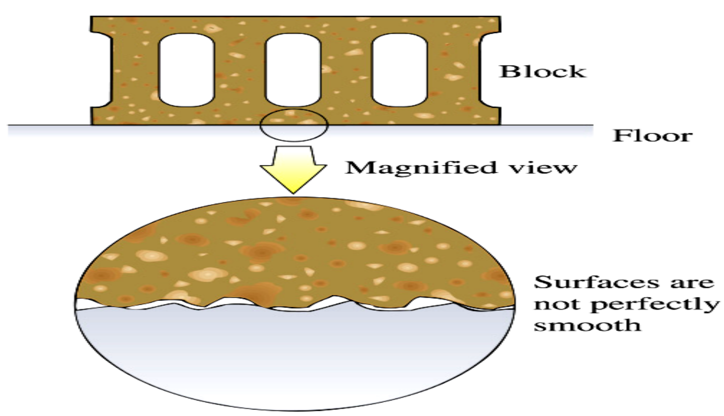
Weld points need to be either initially broken to start the object moving (**static friction**) or broken continuously to maintain the sliding motion (**kinetic friction**).

Experimentally the following features are observed to be true of the force of friction:

1) Friction always **opposes the motion**. The force is **dissipative** and its direction is parallel to the surface of the object in motion.

2) The magnitude of the friction force is **proportional to the objects normal force** $f_s \leq \mu_s N$ for static friction and $f_k = \mu_k N$ for kinetic friction. **μ_s and μ_k** are the **coefficients** of static friction and kinetic friction respectively.

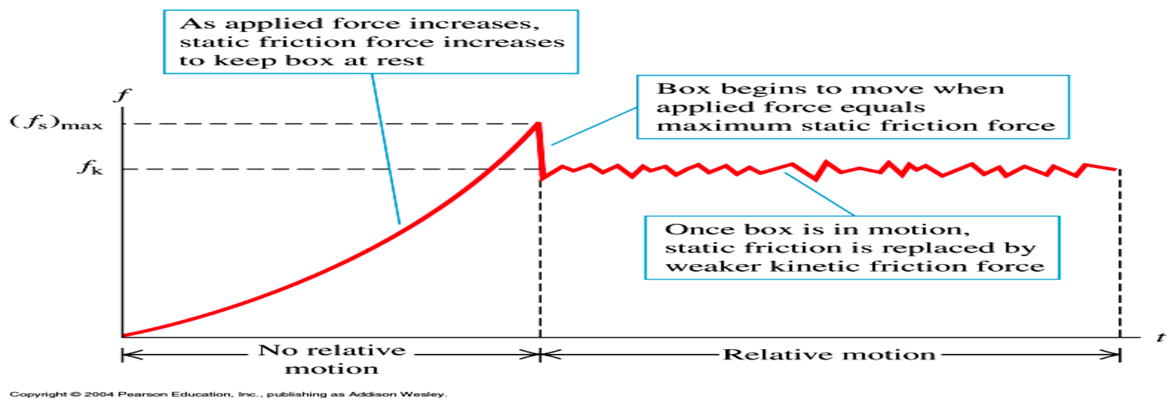
3) **$\mu_s > \mu_k$.** This makes sense since a stationary object forms stronger **contact welds**. The kinetic force of friction is less than the static force of friction.



4) f_s and f_k are **independent of the surface area** of contact and object velocity.

5) When $\vec{F}_{Parralel}$ to the surface exceeds $f_{s,\max} = \mu_s N$ the object breaks free.

6) The magnitude of static friction force is equal to the magnitude of $\vec{F}_{Parallel}$ that is applied up until the object begins to slide.



Drag Force and Terminal Speed:

For objects moving through a **fluid** such as the atmosphere, a friction **drag force** $\vec{D}$ results at the fluid-surface interface.

For low particle velocities when flow is **laminar or streamline**, the drag force on a spherical object of radius r follows **Stokes' Law**: $D = 6\pi\eta r v$

Here η is the fluid **viscosity**, a measure of internal friction in the fluid layers.

For **Reynolds number** $N_R = {2\rho v r}/{\eta} \geq 2000$ fluid flowing across a spherical surface begins to flow **turbulently** and drag force depends on the square of the velocity:

$$D = \frac{1}{2} C \rho A v^2$$

ρ **is fluid density**, A **cross-sectional area**, v **velocity and** C **a drag coefficient**.

By setting $D = mg$ and solving for free fall **terminal velocity**:

$$v_t = \sqrt{\frac{2mg}{C\rho A}} = const. \Rightarrow$$ The object free fall acceleration has ceased.

Sky Diver ➔125 mph **Baseball ➔94 mph**
Ping-Pong Ball ➔20 mph **Parachutist ➔11 mph**