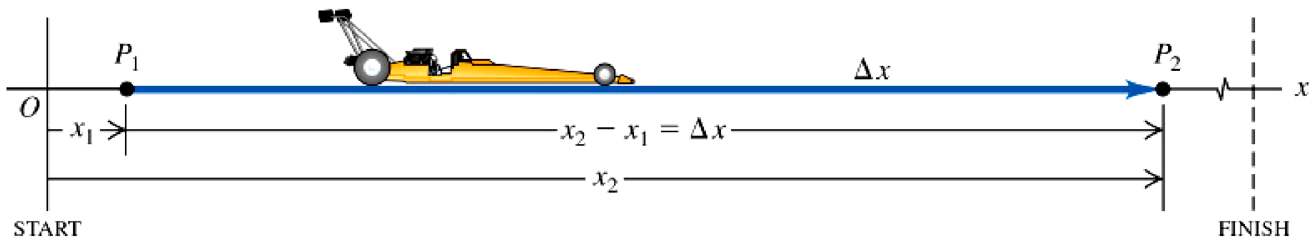
## Kinematics:

The descriptive classification of motion is **Kinematics**. Parameters such as **position**, **velocity**, and **acceleration** are interrelated to describe motion in time without any reference to forces that may be present.

## Displacement:

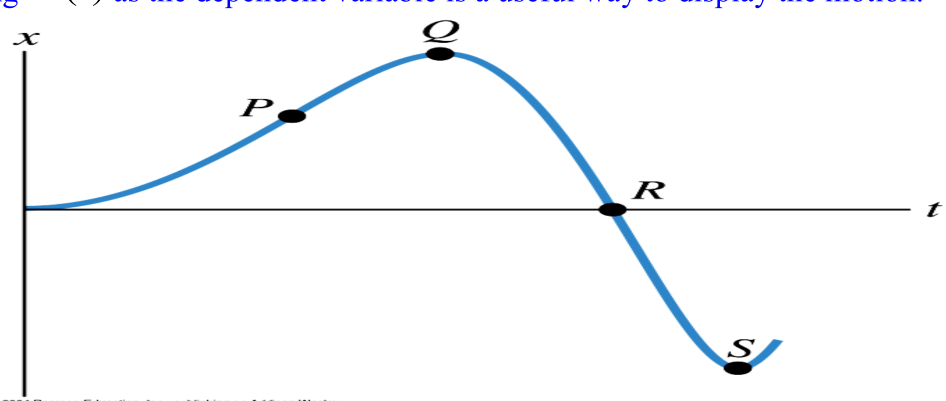
Position in **one dimension** implies a number line and a $\pm$ value relative to some origin. A change in position or displacement is $\Delta X = X_2 - X_1$ and is a **vector** quantity requiring specification of both the $\Delta X$ magnitude and either $+$ or $-$ direction.




In one dimension, $\Delta X < 0 \Rightarrow$ motion toward the origin and $\Delta X > 0 \Rightarrow$ motion away from the origin for $X_2$ & $X_1$ both positive.

If $X_2 = X_1$ then while the total distance a particle has moved may not be zero, its displacement (the vector quantity) is identically zero.

Graphing particle position versus time using time as the independent variable and plotting $X(t)$ as the dependent variable is a useful way to display the motion.

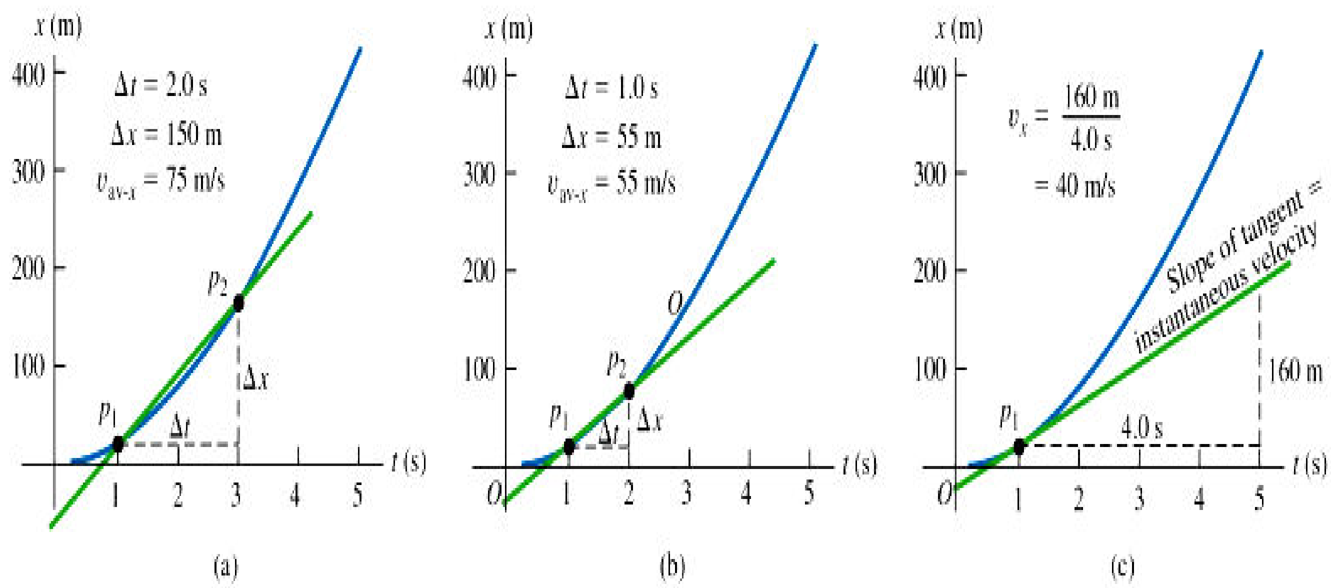


**Velocity**:

Given $X(t)$, the slope of lines connecting two points of the graph $\Delta X / \Delta t$ and the slope of tangent lines to any point along the curve define, respectively, the notions of average velocity and instantaneous velocity:

$v_{av-x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ Slope of the line connecting $(t_2, x_2)$ & $(t_1, x_1)$

$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ Slope of the tangent line at any point $(t, x)$ along the curve.




Since displacement is a vector quantity, velocity is also a **vector** quantity requiring the specification of a magnitude as well as a direction. One-dimensional motion for which

$\Delta X > 0 \Rightarrow v_{av-x} > 0$

A **scalar** quantity (magnitude only) of interest also is the **average speed**. The ratio of total distance traveled to the time elapsed for this travel without deference to direction:

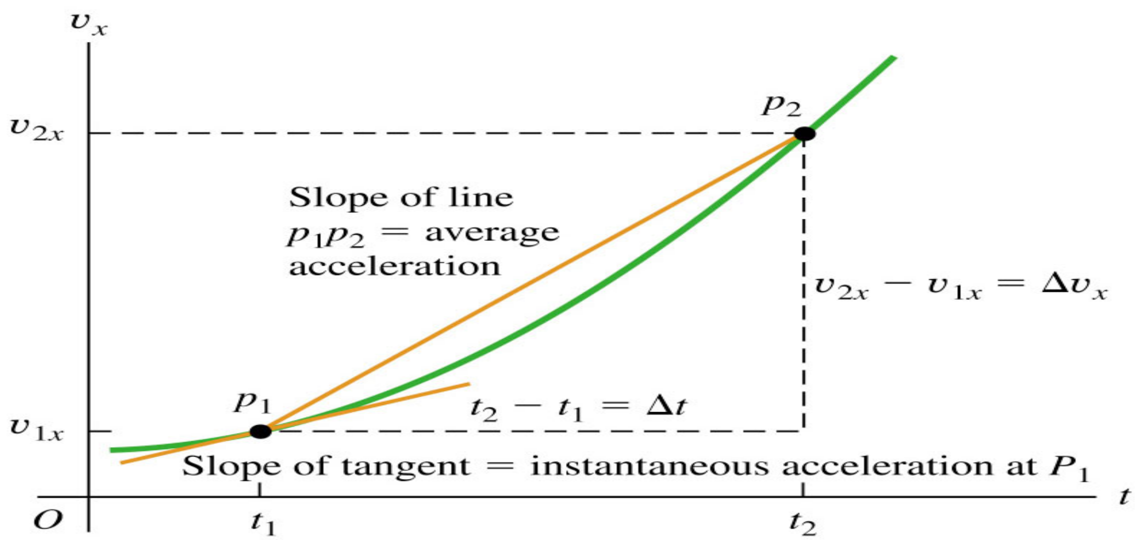
$$S_{avg} = \frac{Total\ \ dis\tan ce}{Total\ \ time}$$

**Acceleration**:

Similarly given a $v(t)$ curve, which graphs particle velocity at any given time during the motion, we can find the slope between two points on the curve or slopes of tangent lines at any point on the curve and define average and instantaneous acceleration:

$$a_{av-x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$ Slope of the line connecting $(t_2, v_2)$ & $(t_1, v_1)$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$ Slope of the tangent line at any point $(t, v)$ along the curve.





Whereas velocity corresponds to how the particle position changes in time, acceleration gives us information about how the **particle velocity changes with time**.

Acceleration is a **vector** quantity. Particle motion depends on the arithmetic signs of acceleration and velocity as follows: If the signs of both acceleration and velocity are the same, for example both positive or both negative, then particle speed will be increasing. However, if acceleration and velocity are of opposite sign then particle speed decreases.

Note the case where $v_{2x} = v_{1x}$ , that is, the **object velocity is constant**, then the acceleration is identically zero and Newton's second law ➔ No external forces.

We assume acceleration if present is constant and the kinematic equations derived below will only be valid in physical situations for which the condition of **constant acceleration** applies.

**Kinematic Equations**:
The kinematic equations are a set of principally three equations that hold true for constant acceleration problems involving particle motion where some or all of the kinematic parameters (position, velocity and acceleration) are known.

Derivable from the definitions of velocity and acceleration, the kinematic equations are:

Let $v_{2x} = v_x$ and $v_{1x} = v_{0x}$

Let $t_2 = t$ and $t_1 = 0$

Then $a_{av-x} = \dfrac{v_x - v_{0x}}{t-0} = a_x$

**Kinematic Equation 1**: $v_x = v_{0x} + a_x t$ **Note linear in the time variable**.

From the average velocity definition: $v_{av-x} = \dfrac{x - x_0}{t-0}$

$$x = x_0 + v_{av-x} t$$

Further since: $v_{av-x} = \dfrac{v_x + v_{0x}}{2} = \dfrac{(v_{0x} + a_x t) + v_{0x}}{2}$

**Kinematic Equation 2**: $x = x_0 + v_{0x} t + \dfrac{1}{2} a_x t^2$ **Quadratic in the time variable**.

$$v_x{}^2 = (v_{0x} + a_x t)^2 = v_{0x}{}^2 + 2 v_{0x} a_x t + a_x{}^2 t^2$$

From equation **2**: $2a_x \Delta x = 2a_x v_{0x} t + a_x{}^2 t^2$

**Kinematic Equation 3**: $v_x{}^2 = v_{0x}{}^2 + 2a_x \Delta x$

**Free Fall Motion**:

In free fall motion, the constant acceleration $a_x$ in the kinematic equations is that acceleration a particle experiences due to the earth's gravitational attraction:

$$a_x = g = 9.80\frac{m}{s^2}$$

Further, since speeds become more negative as an object falls towards the earth (recall velocity is a vector quantity), $g$ enters into the kinematic equations with a minus sign:

**Free Fall Kinematics in the 'Y' coordinate**:

**Kinematic Equation 1**: $v_y = v_{0y} - gt$

**Kinematic Equation 2**: $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

**Kinematic Equation 3**: $v_y{}^2 = v_{0y}{}^2 - 2g\Delta y$