Fluids

For most of the following, we assume **idealized fluids** with the following properties:

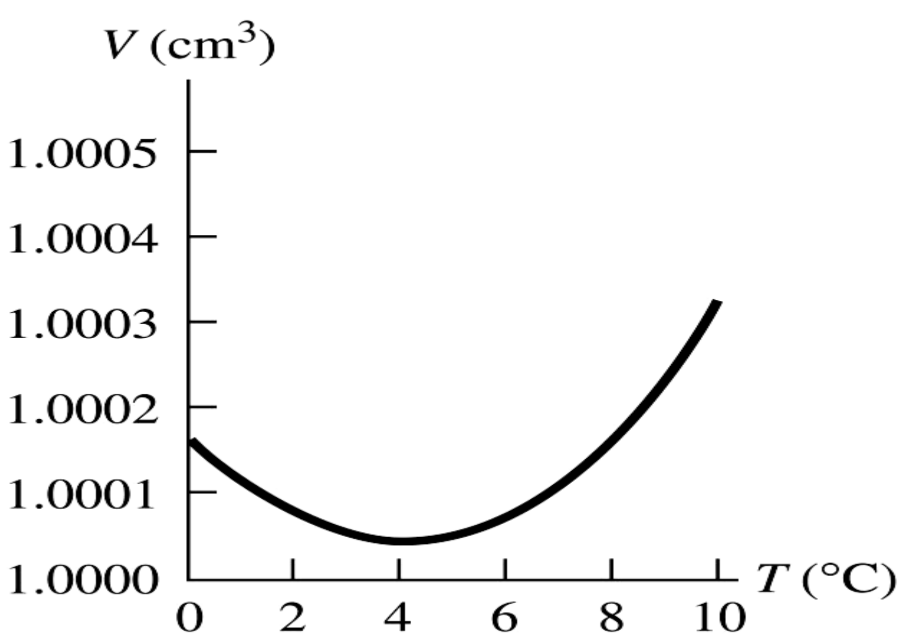
1) **Incompressible → Constant Density.**
2) **Non-viscous → No internal friction…dry water.**
3) **Streamline or laminar flow.**
4) **Irrotational, non-turbulent flow.**
5) **No sources or sinks into or away from the systems.**

Density:

The **density** of a substance is its mass per volume:

$$\rho = \frac{M}{V} \qquad SI\_Units \qquad kg/m^3$$

For most solids, density **increases** upon cooling and contraction. Water is an interesting exception: ρ for water at $4^o C$ is $1000\ kg/m^3$. As temperature drops below $4^o C$, the density of water **decreases!**



Specific gravity is the ratio of the substance density to the density of water at $4^o C$.

$$S.G. = \frac{\rho_{substance}}{\rho_{water\_at\_4^o C}}$$

The units of specific gravity are naught.

Pressure

The pressure exerted on an object of surface area A is:

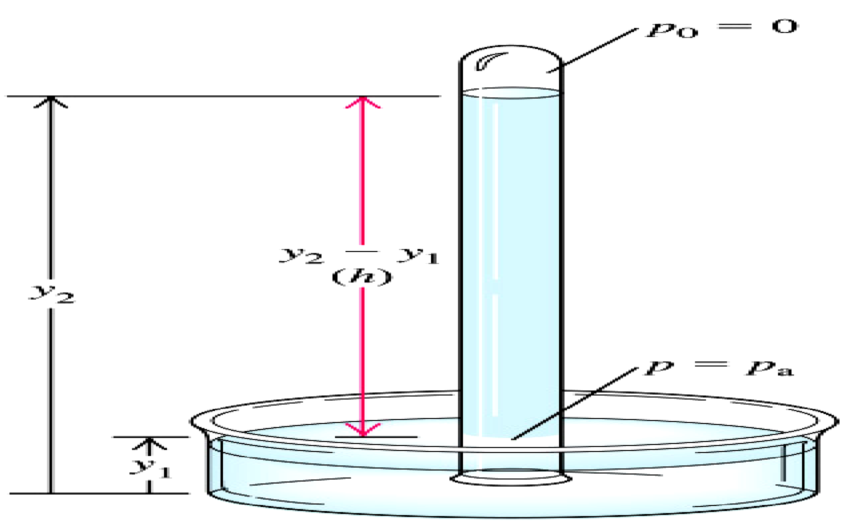
$$p = \frac{F_\perp}{A}$$

$F_\perp$ is force exerted perpendicular to the surface.

SI Units of the scalar pressure are the Pascal. $1Pa = 1 N/m^2$

The **mercury barometer** invented by **Torricelli** in the 1600's measures $1atm$ of pressure as $760mm$ of Hg :

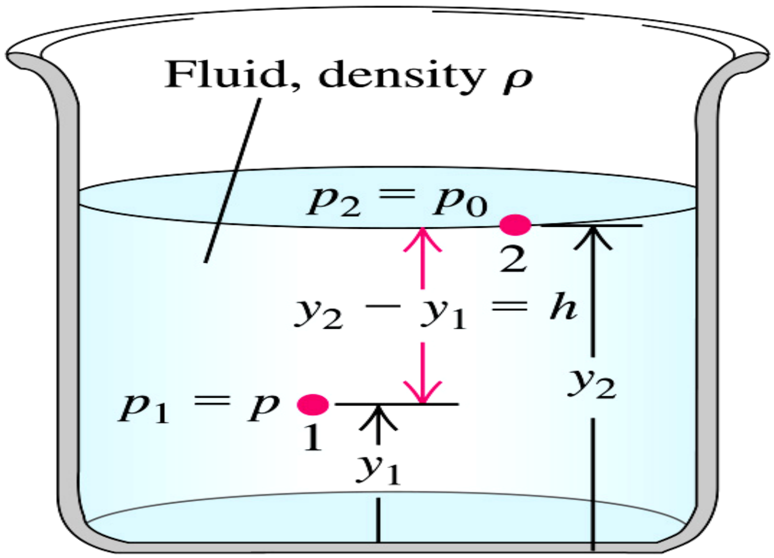
An inverted glass tube with mm scale markings is placed open end down in a bowl of mercury that is exposed to the atmosphere. The top of the tube is evacuated and the Hg rises to a level in the tube resulting from the 'push' of the atmosphere on the Hg in the bowl. This height will be $760mm$ if there is $1atm$ of pressure.



Hydrostatic Pressure

An object is submerged in a **static fluid** at a depth below surface level. The pressure resulting from the fluid weight above the submerged object is the **hydrostatic pressure**:

All sides of the submerged object have equal applied forces since the object is static.



The force from the fluid weight is: $F = Mg = \rho V g$

$V = A*(y_2 - y_1)$ is the volume of fluid on top of the submerged object.

For any depth h then, the **hydrostatic pressure** will be:

$$P = F/A = Mg/A = \rho Vg/A = \rho gh$$

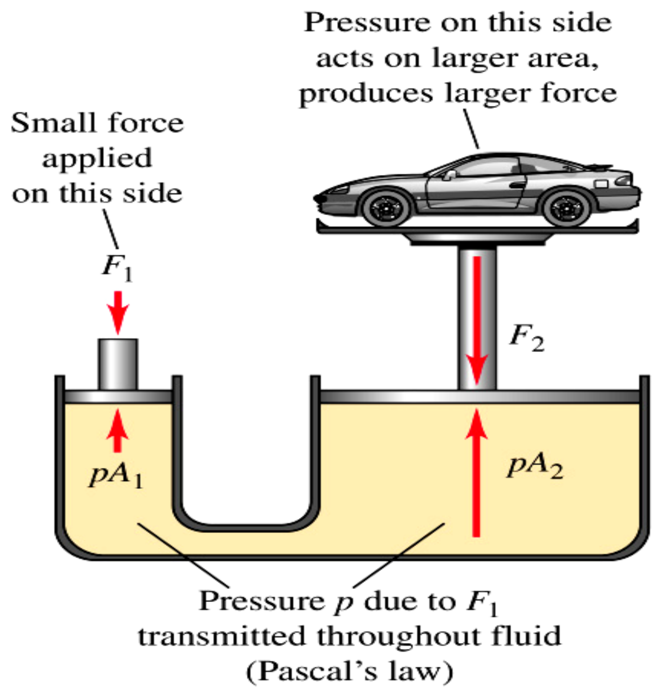
The **Absolute Pressure** on the submerged object is $p = p_0 + \rho gh$.

A pressure **gauge** automatically subtracts off an atmosphere of pressure from its readout meaning that $p_{Gauge} = p_{Absolute} - 1atm$

Pascal's Principle

Incompressibility of an ideal fluid can be used to gain mechanical advantage with hydraulic machinery using Pascal's Principle:

Since an ideal fluid is incompressible, an applied force at one location in the fluid will result in a pressure change **transmitted undiminished** throughout the entire fluid.



$$F_2 = pA_2 = F_1 \frac{A_2}{A_1}$$

Amplification of the input force.

The price to be paid for this advantage is seen by examining the energetics:

$$W_{in} = W_{out} \qquad F_1 d_{in} = F_2 d_{out}$$

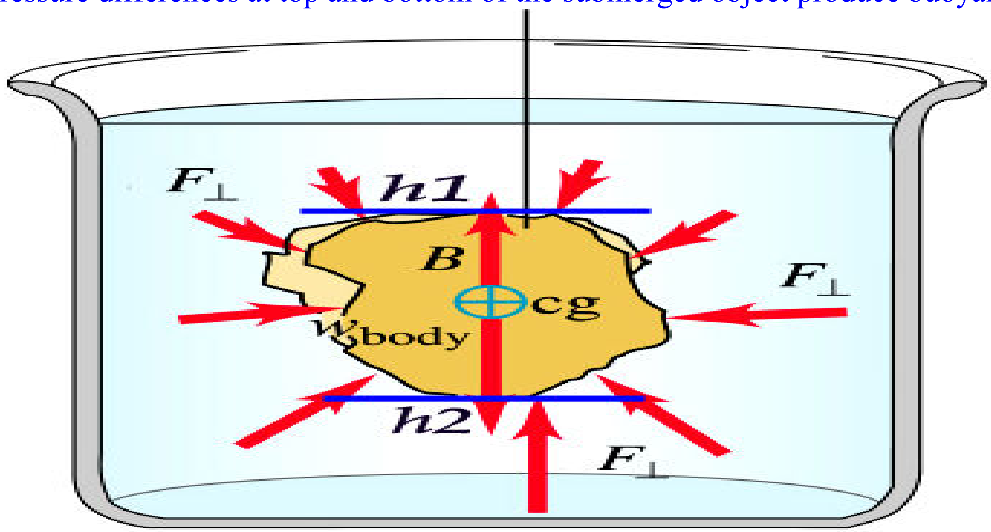
$$d_{out} = \frac{F_1}{F_2} d_{in} = \frac{A_1}{A_2} d_{in}$$

Only a fraction of the input distance is recovered in the output movement.

Archimedes' Principle

Archimedes determined that the **buoyant force** on an object partially or totally submerged in a fluid is equal in magnitude to the **weight of the fluid displaced**.

Pressure differences at top and bottom of the submerged object produce buoyancy:



Hydrostatic pressure at h_1 is $\rho g h_1$ and at level h_2 , the pressure is $\rho g h_2$.

The difference is $\Delta P = \rho g (h_2 - h_1)$ leading to a buoyancy force of:

$$B = \Delta P \bullet A = \rho g (h_2 - h_1) \bullet A = \frac{M_{fluid}}{V_{fluid}} g (V_{object})$$

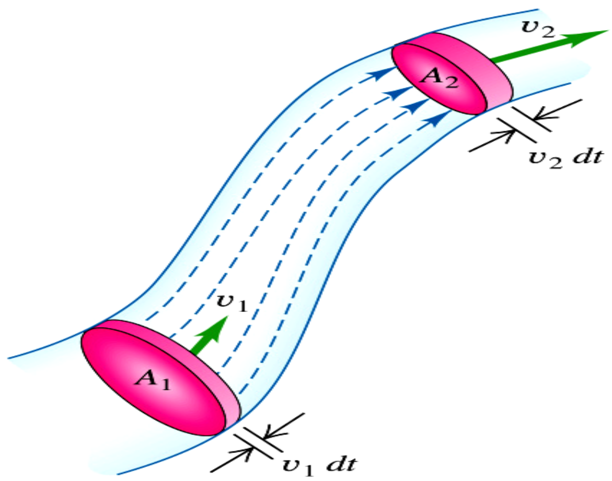
Since the objects volume is equal to the fluid volume displaced $B = M_{fluid} g$.

Submerged objects have an apparent weight: $W_{apparent} = W_{actual} - B$

Sinking or floating becomes a question of which is greater the object weight or its buoyant force: $\rho_{object} V_{object} g > or < \rho_{fluid} V_{fluid} g$ Since the object volume equals the fluid volume displaced $\rho_{object} > \rho_{fluid} \Rightarrow Sinking$ and $\rho_{object} < \rho_{fluid} \Rightarrow Floating$

Fluid Dynamics

Beginning with the **equation of continuity**, consider the streamline flow of an ideal fluid through a pipe of varying cross-sectional area and conserve fluid mass as follows:



Mass conservation ➔ $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

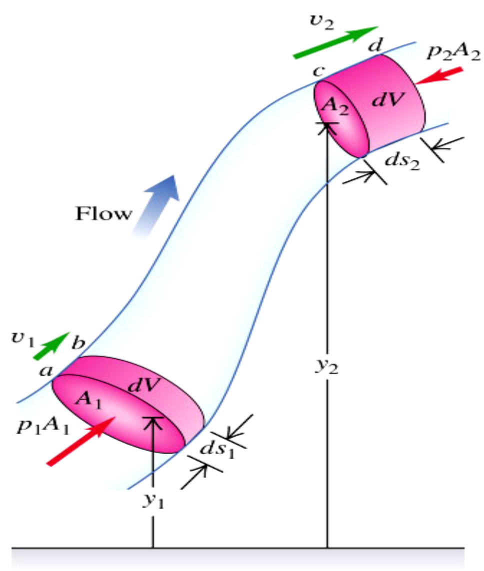
If density is a constant at location 1 and 2, then:

$$A_1 V_1 = A_2 V_2$$

When A_2 constricts the speed of flow increases as $V_2 = A_1 V_1 / A_2$.

Check a garden hose for experimental verification.

From the Work-Energy Theorem **Bernoulli's Equation** for ideal fluids is:



A pressure at A_1 implies work done on the fluid $W_1 = p_1 A_1 ds_1$

Countering this is the pressure at A_2 that does work $W_2 = p_2 A_2 ds_2$

The Work-Energy Theorem is $W_{Net} = \Delta KE + \Delta PE$

$$p_1 A_1 ds_1 - p_2 A_2 ds_2 = \frac{1}{2}\rho V {v_2}^2 - \frac{1}{2}\rho V {v_1}^2 + \rho V g y_2 - \rho V g y_1$$

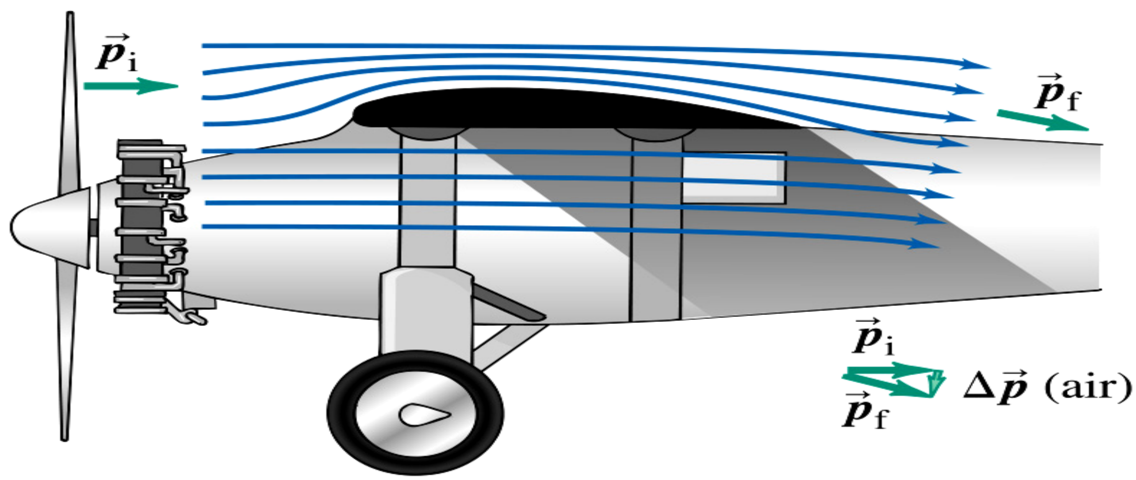
$$p_1 - p_2 = \frac{1}{2}\rho {v_2}^2 - \frac{1}{2}\rho {v_1}^2 + \rho g y_2 - \rho g y_1$$

$$p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2 = p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1$$

This quantity has a constant value on any streamline in the fluid.

Dynamic lift on an airplane wing is as follows:

Streamlined air forced over the top of the wing travels a greater distance over the top of the wing before rejoining the streamline at the back of the wing. Fluid velocity on top of the wing is increased and this decreases pressure topside.



Taking $y_1 \cong y_2$ in Bernoulli's Equation the result is

$$p_2 + \frac{1}{2}\rho v_2^{\,2} = p_1 + \frac{1}{2}\rho v_1^{\,2}$$

$v_2 > v_1 \Rightarrow p_2 < p_1$ and the condition for lift is established.

Another less innocuous application is **arteriosclerosis** and the resulting **vascular flutter**. The sequence of events results in blood flow problems:

Plaque in artery walls $\Rightarrow$ constriction $\Rightarrow$ reduction in cross-section area $\Rightarrow$ increase in blood velocity $\Rightarrow$ reduced pressure per Bernoulli Eqn. $\Rightarrow$ collapse of artery $\Rightarrow$ no blood flow $\Rightarrow$ artery reopens $\Rightarrow$ repeat of cycle $\Rightarrow$ fluttering $\Rightarrow$ trouble.

With an **aneurysm**, a weak artery wall $\Rightarrow$ increasing cross-section area $\Rightarrow$ reduced blood velocity $\Rightarrow$ increased pressure per Bernoulli Eqn. $\Rightarrow$ rupture $\Rightarrow$ trouble.

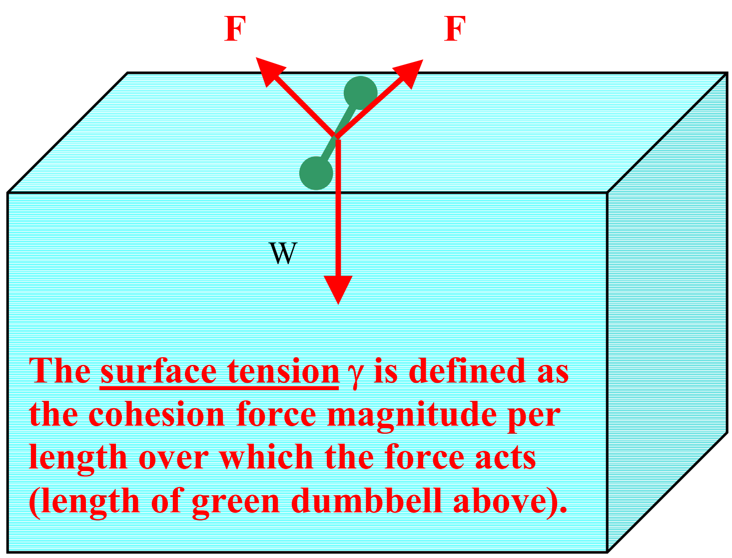
Surface Tension, Capillarity, and Viscosity

At the surface of a liquid, there exist an outward force resulting from molecular collisions **beneath** the surface.

Balancing these forces are **cohesive** attractions between surface molecules with molecules just below the surface.

The net effect produced is a **surface tension** from a stretched membrane-like surface layer of the liquid molecules configured in their most energetically favorable state that corresponding to the least amount of surface area. [E.g., Spherical (not cubic) raindrops]

For a small object at rest on the surface of a liquid, an upward tension balances the objects weight as restoration of the minimal area equilibrium condition is sought.



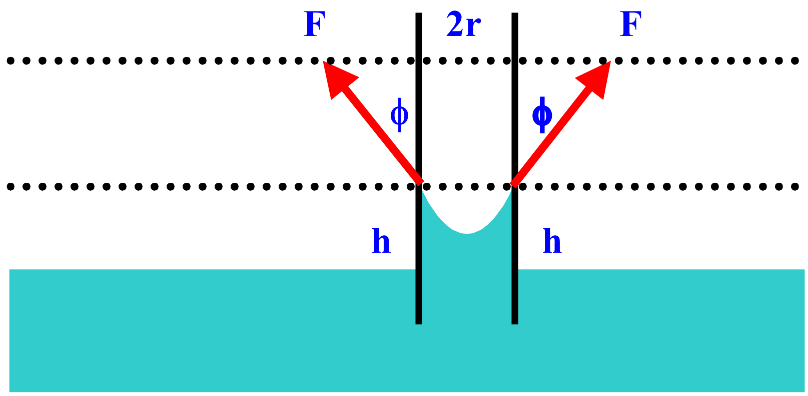
$$\gamma = F/L$$

SI units of J/m^2

Capillarity

For **capillarity,** consider the liquid-container interface where interaction between liquid molecules and container molecules are occurring. This **adhesive** interaction is either greater than or less than the cohesive force between the liquid molecules and, depending on which is the case, capillary tubes will either 'draw' the fluid upward as it is placed in contact (adhesive forces greater than cohesive forces) or provide a border around which the fluid can increasingly decrease its surface area.

In terms of a **contact angle** ϕ a result for the height to which capillarity pulls liquid above a reservoir level is:



$$F = \gamma L = \gamma 2\pi r$$

$$F Cos(\phi) = mg$$

$$\gamma 2\pi r Cos(\phi) = \rho V g = \rho \pi r^2 h g$$

$$h = \frac{2\gamma Cos(\phi)}{\rho r g}$$

Capillary tubes are used to draw small quantities of a fluid that is either difficult to access or in small supply.

Viscosity / Poiseuille's Law / Reynolds Number

Viscosity refers to internal friction between layers of a flowing fluid and between the fluid layers and other materials such as pipes. The speed of a viscous fluid within a pipe is usually zero at the pipe wall and maximal at the center of the pipe.

The **coefficient of viscosity** η is a constant for **Newtonian Fluids** and is inversely proportional to the flow rate as in **Poiseuille's Law**:

Poiseuille's Law is:

$$\frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}$$

L Is the length of pipe and $P_1 - P_2$ is the pressure differential.

The SI units of viscosity are $\frac{N \bullet s}{m^2}$ **The CGS unit is the Poise** $\frac{Dyne \bullet s}{cm^2}$

Reynolds Number

In characterizing motion of viscous fluids, the ratio of **dynamic pressure** $\frac{1}{2}\rho v^2$ to the **viscous stress** $\frac{\eta v}{d}$ gives a quantity proportional to the **Reynolds number**.

$$N_R = 2 * \left\{ \frac{1}{2}\rho v^2 \Big/ \frac{\eta v}{d} \right\} = \frac{\rho v d}{\eta}$$

Where d is piping diameter for example.

For **Reynolds number** $N_R = {}^{2\rho v r}\!/_{\eta} \geq 2000$ fluid flowing across a spherical surface begins to flow **turbulently** and drag forces depend on the velocity squared.

For turbulent flow, the solutions to the fluid equations of motion are unstable and evolution depends strongly on **initial conditions**.

At low particle velocities when flow is **laminar or streamline**, drag force on a spherical object of radius r follows **Stokes' Law**: $D = 6\pi\eta r v$

Transport Phenomena and Fick's Law

Fluid flow within another medium results when concentration gradients exist between two points. A directed flow of transported fluid molecules from regions of higher concentration to regions of lower concentration takes place until equilibrium is established.

Fick's Law quantifies diffusion processes:

$$\frac{\Delta M}{\Delta t} = \frac{DA(C_2 - C_1)}{L}$$

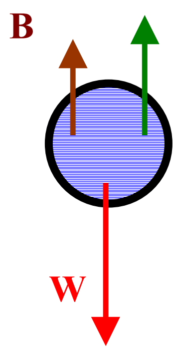
The mass diffusion rate.

Here $C_2 - C_1$ is the concentration gradient and D the diffusion coefficient for the diffusing fluid/medium system.

Sedimentation / Centrifugation

Sedimentation processes involving spherically shaped objects and taking place at a low Reynolds number have a drag force on the object according to $F = 6\pi\eta a v$.

Balancing weight against buoyant and drag forces gives us the terminal velocity:



$$F = 6\pi\eta a v$$

$$mg = B + 6\pi\eta a v = B + kv_t = \rho_f V_{object} g + kv_t$$

$$mg = \rho_f \frac{m}{\rho_o} g + kv_t$$

$$mg(1 - \frac{\rho_f}{\rho_o}) = kv_t$$

$$v_t = \frac{mg}{k}(1 - \frac{\rho_f}{\rho_o})$$ For sedimentation in low Reynolds number fluids.

In a **centrifuge**, the gravitation constant g is replaced by the centrifugal acceleration $v^2 / r = \omega^2 r$ and the result:

$$v_t = \frac{m\omega^2 r}{k}(1 - \frac{\rho_f}{\rho_o})$$ And more massive particles with greater v_t settle first.